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> Mariola SANCHEZ ROMERO
Miguel Hernández University

>Amparo URBANO SALVADOR
University of Valencia

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Consumers' privacy concerns and price setting in a multi-channel monopoly

Mariola Sánchez-Romero* and Amparo Urbano†

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Abstract

This paper presents a dynamic model of price setting under market signaling and learning. There is a monopolist selling a good in two purchasing channels - traditional market and Internet -. Consumers buying in the online market have a privacy concern, that is unknown to them when they make their purchase in the first period. The monopolist receives a noise signal about the consumers' average privacy, which allows her to adjust the price in both sale channels. The prices designed by the monopolist in the second period will serve as a signal to consumers, and this, together with their experience from the first period, will determine their second period demand. The paper shows, on the one hand, how the monopolist set prices and uses them to signal her private information to the consumers. This strategy allows her to price discriminate between the two different purchase channels and obtain the consumers' maximum willingness to pay. On the other hand, the paper offers a new reason for multi-channel market price dispersion which depends on the population-average privacy concerns.

Keywords: Privacy concerns, Learning, Market channels, Signaling equilibrium

JEL Classification: D83, D42, C72.

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* *Corresponding author: Mariola Sánchez Romero, Department of Economics and Financial Studies, Miguel Hernández University. Avda. Universidad, s/n., E-03202 Elche. email: m.sanchezr@umh.es.* The author thanks the financial support from the Spanish Ministry of Economy, Industry and Competitiveness (BES-2014-069278).

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1 Introduction

We all live in a networked society, where we perform a set of routine activities thanks to our devices and different applications that allow online shopping, communication and social relations, access to global information instantly, geolocations, etc. Lately, different media point out the great public exhibition to which the new digital age obliges us. Many news emphasizes the vulnerability in privacy that this display entails, even questioning devices that resort to facial recognition, that is, the ability to read faces. This fact, consequently, has been developing privacy concerns in the whole society where privacy and its definition has become a moving target over time, difficult to specify, and in expensive treasure to cherish. In words of Danah Boyd, "The balance of forces has shifted in the networked age. People are now public by default and private by effort".

The online presence of companies has become a strategic necessity, creating, therefore, great opportunities and challenges for them thanks to the rapid development of information technology. Given the economic interests, a new personal data market has emerged creating new actors, such as data brokers, that collect personal information about consumers, and sells that information to other organizations. For that individual who still wonders how online companies should generate revenues the answer is simple: "(...) That of a web where everything is free, but we pay for it through our privacy". The existence of privacy concerns can affect consumer behavior in a digital environment. Although 69% of the Internet users in the European Union shopped online in 2018 according to Eurostat, most of them avoid purchasing online because of security matters. France, Norway, Sweden, Finland and North Macedonia present a high percentage of people who avoid purchasing online, more than 22%. They are followed by countries where the percentage presents values between $(16\% - 22\%)$, among them, Portugal, Denmark, Spain and Latvia (Eurostat, 2019). The most important fact is that this is not a negligible percentage of individuals, and should be taken into consideration by e-commerce firms when they draw their retail's strategies.

New ways to reach consumers and their needs are now possible as a consequence of the fast growth of technology. Thus, new business models have emerged and therefore, firms have been switching to multi-channel strategies to adapt to this new regime and age. Even though that one can think about firms opening online shops is the new trend, the other way round is also starting to happen. For example, Amazon and AliExpress, large e-commerce companies in the world, have recently surprised the world with the news of opening physical stores.[1,2] Everything suggests that the multichannel strategy will mark the strategic design of retail companies in the future characterized by different channel types, relationships, and structures. In words of Gorodnichenko et al. (2018), there is a potential complementary of online and offline shopping and therefore, companies would increase their profits if they had a presence on several channels.

However, companies may face different difficulties with pricing under this new retailing strategies. Indeed, recent empirical studies on pricing behavior across channels show conflicting results. Some of them reveal similar prices between multichannel retailers' online and offline channels and little within-retailer price dispersion (Flores and Sun 2014; Cavallo 2017); others indicate that up to 60% of multi-channel retailers engage in channel-based price differentiation and that this trend is increasing (Wolk and Ebling 2010). On the other hand, empirical results also fail to explain the reasons why exist price dispersions. Several possible explanations that could shed light on this matter involve price dispersion based on demographic self-selection and shopping intent (Cuellar and Brunamonti 2014) or the perceived risk in the online channel (Wolk and Ebling 2010). Cavallo (2017) also analyzes price dispersion based on IP addresses or browsing habits (very controversial causes), but surprisingly, they do not find any evidence to support those causes.

We wish to study how privacy concerns affect the prices setting of a monopolist over two purchase channels in two periods of time. Our model is a two-period signaling game where a monopolist and consumers learn from market signals the privacy concerns of the latter. We apply the classical signaling games framework to analyze the informational content of prices and the market performance under incomplete information and privacy concerns.

Our contribution is twofold. On the one hand, we present a dynamic model of signal extraction of consumer behavior and learning. There is a monopolist selling a good in two purchasing channels - traditional market or the Internet -. Consumers buying in the online market have a privacy concern, that is unknown to them when they make their purchase in the first period. In addition, the monopolist receives a noise signal about the consumers' average privacy, which allows her to adjust the price in both sale channels. The prices designed by the monopolist in the second period will serve as a signal to the consumers about the use of

[1]More information in: https://www.forbes.com/sites/annaschaverien/2018/12/29/amazon-online-offline-store-retail/

[2]More information in: https://www.forbes.com/sites/victoriapavlova/2019/08/26/heres-what-aliexpress-madrid-store-could-mean-for-its-european-ambitions/

their privacy, and this, together with their experience from the first period, will determine their second period demand. The paper shows how the monopolist set prices and uses them to signal her private information to consumers.

On the othe hand, we offer a new reason for multichannel market price dispersion in the information age, where new business models and the increase of technology let companies sell their products across online and offline stores (Brick-and-Click channels). This is due to the introduction of a novel aspect in those consumers purchasing online, which is their heterogeneous privacy concerns that affect their willingness-to-pay for the product. Based on the spread of news that relates to privacy, and the importance given to the "privacy paradox" concerning the difference between attitudes and actual behaviors of individuals, it might be natural to assume that consumers may present heterogeneity in their value for their privacy (Acquisti and Grossklags 2004). Namely, consumers have idiosyncratic privacy concerns that evolve over time.

We find both price discrimination and market price dispersion in a dual-channel context. Firstly, the monopolist faces two expected demands, whose difference is due to the existence of privacy concerns that affect the online's demand. This difference reflects the decrease of consumers' confidence to purchasing on the internet due to private information's security. We study the two standard price settings for the monopolist, namely, uniform pricing and price dispersion, both under uncertainty and market learning. Secondly, we compare the two price setting and conclude that the monopolist's best strategy is to price discriminate, establishing different prices among channels. Finally, we analyze how the different channel-prices are related with each other and with the uniform price, finding price dispersion among them.

2 Literature Review

Our work is primarily related to three streams of research. The first stream of literature is related to is privacy. Matters related with privacy and economics is not something new. Recent studies have focused primarily on the protection of information about consumer's preferences or type, and the relationship between privacy and pricing. For a complete survey and to check out the evolution over decades, see Acquisti et al. (2016). Their work review the theoretical and empirical economic literature investigating individual and societal trade-off with sharing and protecting personal data. They consider that privacy sensitivities "are subjective and idiosyncratic, because what constitutes sensitive information differs across individuals" and that is our focus with this article. Villas-Boas (2014) and Chen and Zhang (2009) study price for information strategies in dynamic models, where firms price less aggressively in the first period in order to learn more about their customers and price discriminate in later periods. Acquisti and Varian (2005) and Conitzer et al. (2012) in which merchants have access to "tracking" technologies and consumers have access to "anonymizing" (or record-erasing) technologies, and show that welfare can be non-monotonic in the degree of privacy. In Belleflamme and Vergote (2016) a monopolist has also access to "tracking" technologies but with different grades of tracking, and consumers have access to privacy with a cost. They show that the use of a hiding technology harm those consumers that do not hide, because of the increase in the level of prices due to the "hidders". We do not model privacy as a cost to the customer; our approach is that the concern of privacy is something idiosyncratic for the consumer in a signaling model a là Judd and Riordan (1994).

The second stream of literature examines the decision-making in a context of dual-channel distribution. Although our work is positioned in the literature on dual channels distribution and operations, we do not focus on the aspects commonly taken in this specific field of literature. For many of them, the figure of the manufacturer is not the same as the retailer, and study the strategic relationship between them and their effects on dual channels' prices, profits, variety of products, etc. Xiao et al. (2014) develop a retailer-Stackelberg pricing model to investigate manufacturers' product variety and channel structure strategies in a circular spatial market; Chiang et al. (2003) elaborate a consumer choice model and studied a pricing game involving a manufacturer and a retailer in a dual-channel supply chain. Focusing on the study of consumer behavior, we analyze a context similar to that of Fruchter and Tapiero (2005) in the sense that consumers are heterogeneous in their virtual acceptance, and derive utility according to the channel they choose. In the same line, Chiang et al. (2003) assume that consumers have a lower valuation for the product purchased online than for that bought in the physical channel. Li et al. (2015a) also made such an assumption because the consumers have a lower acceptance for the online channel. In a particular way, this idea is also captured in our model because we assume a willingness to pay known to all participants in the market, the only difference is that consumers derive some uselessness for the purchase in the online channel, and therefore, they will derive in a propensity to pay less, depending on the accuracy in the information and the previous experience. Consumer shopping experience has also been incorporated as an important part in decision-making. Li et al. (2015b) study the appropriate distribution channel given assortment (breadth, depth, prices of assortment), logistic (inventory cost, delivery cost, delivery time) and consumers characteristics. Ofek et al. (2011) incorporates other variables that can alter the consumer behavior, such as shopping trip cost or the consumer cost of returning a mismatched product.

The third stream of literature we are related to is price discrimination and price dispersion literature. In a channel context, consumers derive different utility from various distribution channels (Chu et al. 2007), which, in turn, leads to differences in channel valuations. Indeed, a previous study finds that the willingness to pay for a product purchased through an offline channel can be 8%–22% higher than the willingness to pay for a product purchased through a online channel (Kacen et al. 2013). This fact recognizes the great opportunity to increase profits and engage in channel-based price differentiation (Wolk and Ebling 2010; Khan and Jain 2005), which is also feasible and should encourage companies to pursue a price differentiation strategy whenever possible. We support those findings of price dispersion and price differentiation among channels.

Our results emphasize the importance of consumers' privacy concerns in decision-making in a dual-channel context. This setting aims to give insight on the important role that privacy can have on prices and on the monopolist's optimal strategy. In particular, our findings point out that the monopolist gets higher profits if she discriminates over channels, and sets different prices in a market with signals. Furthermore, it exists price dispersion among channels and the uniform price, and online prices can be higher or lower than both the offline and uniform prices depending on the average privacy concerns in the market. Thus, consumers' privacy concerns may be a relevant explanation to the existence of price dispersion.

The paper is organized as follows: Section 3 explains the general and benchmark model and Section 4 the beliefs updating. Sections 5 and 6 analyze the monopolist's two main strategies to set prices. Section 7 compares the two pricing setting scenarios, analyze whether prices are similar over channels and the uniform price and relates their differences with the market average privacy. Section 8 presents the main highlights and final remarks.

3 The Model

Our model is a two-period signaling game where a monopolist and consumers learn from market signals the privacy concerns of the latter. We apply the classical signaling games framework to analyze the informational content of prices and the market performance under incomplete information and privacy concerns.

The monopolist has an overall demand composed by consumers purchasing from two channels, the traditional channel (the brick and mortar channel, the brick, in short) and the Internet one (the click channel). All consumers know their willingness to pay -it is a manner to say that the product is not new and they are familiarized with its quality or taste- but they may have an element that diminishes their utility i.e., their privacy concerns. We take a linear-quadratic approach to make the analysis tractable and provide close form solutions, which facilitate comparisons. This model is a simple one to begin to analyze market signaling under consumers' idiosyncratic privacy concerns.

We assume that if individual i decides to purchase through the brick channel, then there will not be concerns for privacy. That is, we assume that the traditional channel does not represent any threat to consumers about the usage of their personal information. Let q_{it} be consumer i's demand in period t. Then, the demand in the brick channel in period t is,

$$q_{it} = \theta_i - p_t, \tag{1}$$

where p_t is the price in period t. As mentioned above, consumers know their willingness to pay for the product represented by θ_i. Therefore, consumer i would be willing to pay $q_{it}\theta_i - \frac{{q_{it}}^2}{2}$ for q_{it}. However, if individual i decides to purchase through the click channel he does not know their precise privacy concerns, represented by α_{it}, at the time of the purchase. Thus, consumer i's demand in the click channel in period t is

$$E\left\{\theta_i - \alpha_{it} - p_t|\Omega_{it}\right\}, \tag{2}$$

where Ω_{it} is consumer i's information at time t. The privacy to an individual i who decides to purchase a product in period $(t = 1, 2)$ through the online channel is represented by an index α_{it}, equal to

$$\alpha_{it} = \tilde{x} + \widetilde{\omega_i} + \widetilde{v_{it}}. \tag{3}$$

Random variable $\tilde{x}$, $\widetilde{\omega_i}$ and $\widetilde{v_{it}}$ represent the population-average privacy in that specific product market, the individual i's persistent deviation from that population average privacy and his specific-time deviation, respectively. The random variables have the following distributions $\widetilde{\omega_i} \sim N\left(0, \sigma_\omega^2\right)$, $\widetilde{v_{it}} \sim N\left(0, \sigma_v^2\right)$ and $\tilde{x} \sim N\left(\bar{x}, \sigma_x^2\right)$. Therefore, $E\left\{\tilde{\omega}\right\} = E\left\{\tilde{v_{it}}\right\} = 0$. We also assume that they are all normally and independently distributed. Normality has the unpleasant feature of an unbounded support, allowing the possibilities of negative demand and prices. On the other hand, normality has the highly desirable feature of implying linear updating rules for consumers, which simplifies our analysis considerably.

Variable $\tilde{\omega_i}$ catches up differences between consumers. Of course, some consumers do not care about privacy at all. However, some others consumers may consider privacy policy of vital importance and, in particular, if a consumer detects that some private information is used in a harmful way, it will increment the value of α_{it} and hence will lower the utility of this channel. Variable $\tilde{v_{it}}$ is a external shock. For example, there may be an official announcement about a new privacy policy or a new security system that permit consumers to avoid being followed by cookies.

Moreover, we study a context where consumers' demand of products is positive. Thus, the willingness-to-pay for the product, θ_i, in the brick channel is large enough in order to have a positive demand in period 1 and 2, i.e., $\theta_i > p_t$. Furthermore, in the same way, in the click channel, in period $t = 1, 2$ we assume that the willingness to pay is higher than the expected privacy concerns with respect to the set of information in each period t, i.e., $\theta_i > E\{\alpha_{i1}|\Omega_{i1}\}+p_1$ in period 1, and $\theta_i > E\{\alpha_{i2}|\Omega_{i2}\} + p_2$ in period 2.

We assume that privacy concerns are something private or individual to each consumer. Furthermore, the fact that the willingness to pay is equal and known between channels allows us to focus on the privacy concerns as distorting element of the market equilibrium analysis. Namely, we study how privacy concerns can influence the consumer's purchasing behavior, which, in turns, influences the monopolist price setting behavior in both channels.

It is also assumed that the monopolist can get some type of extra mark-up on the price charged in the online market depending on the units sold through this specific channel. For example, she can sell the private information regarding consumers' data using this channel and get some extra profit. This mark-up is denoted by $r \in (0, 1)$, with $r = 0$ meaning that she does not sell the information and, therefore, does not get any extra mark-up. Thus, $r = 1$ represents the case when the monopolist sells information and the mark up is a total percentage over the price.

The firm receives a private signal about consumers' average privacy concerns after period 1. In addition, if consumers buy on the Internet, the monopolist will have another signal about the population-average privacy operating on this channel. The signal received by the monopolist is

$$z = \tilde{x} + \tilde{\varphi}, \tag{4}$$

where $\tilde{x}$ represents the same random variable showing, as before, the average privacy in the market, and $\tilde{\varphi}$ is an external shock which is distributed normally $\tilde{\varphi} \sim N\left(0, \sigma_{\varphi}^2\right)$. Information about privacy concerns is important to the monopolist since she will be able to storage consumers' personal data, selling them to a third party or use this information in her interest to price discriminate. Therefore, signal z represents an important information to the monopolist's second period action and it will be observed after first-period sales. With this particular definition of z we can now give a more complete interpretation of $\tilde{x}$ and the random variable $\tilde{\varphi}$. Thus, $\tilde{x}$ is the portion of the mean effect on the population which is detectable through z. Therefore, if $\tilde{x}$ is independent and not correlated with $\tilde{\varphi}$, then the monopolit's private signal, z, will signal exactly the actual average population privacy concerns of consumers purchasing in the online channel. If $\tilde{\varphi}$ were correlated, then $\tilde{x}$ would not be the average privacy concerns about using this specific channel, but its ex-ante expectation.

The overall sales of the monopolist come from the two channels. Let the parameter λ represent the sales coming from the brick channel and $(1-\lambda)$ the proportion of sales from the online channel. We assume that λ is exogenous and the total mass of consumers is normalized to 1.[3] Therefore $\lambda \in (0,1)$.

We also assume that the unit production cost in each period is common knowledge and normalized to zero. The timing of the game is as follows:

In period 1, the market for the product opens. The monopolist has to decide her price strategy -whether to practice price discrimination or not- for both channels and announce the first-period price(s). In this first period, there is no information generated by any player i.e., there is nor private information for the monopolist neither learning by the customers. Therefore, information set Ω_1 consists of simple expectations: the monopolist has an expected demand from the online channel, Ω_{m1}, where m indicates the set of the monopolist's information. Representative consumer $i's$ who purchases in the click channel, has an expected privacy concern and his set of information is given by Ω_{i1}. These consumers observe the market price and decide how much to purchase of the product given their expectations on privacy concern. The remaining consumers observe the market price and buy in the traditional -brick- channel.

Note that at the beginning of the first period, consumers of the click channel are uncertain about their concerns on privacy and need some experience to update their information. Since it is common knowledge that the monopolist will receive a private signal about the privacy mean

[3] We assume λ fixed in order to obtain close form solutions. If λ were not fixed, we should specify it as a function depending on prices.

at the end of period 1, then at the beginning of period 2, both consumers and the firm will have some new information.

In period 2, the set of information is Ω_2. First, the firm observes a private signal about the average privacy concern on the online channel, $z = \tilde{x} + \tilde{\varphi}$, and first period purchases. Both elements constitute her information set in $t = 2$, Ω_{m2}, and then, the firm announces her period 2 price schedule. Second, consumers learn about their real concerns for privacy from their purchases in the first period and from the second period price -they are able to make an inference on z through the market price- and finally, they make a decision. Therefore, the consumers' information set, Ω_{i2}, consists of their previous purchase experience, α_{i1} and the inference made over z from the second period price, once this price is announced.

The above two-period game with incomplete information is a dynamic Bayesian game. In addition, given that consumers signal their (probabilistic) knowledge about their privacy concerns through their demands, and the monopolist signals her information on consumers' privacy concerns through the second period price, the incomplete information dynamic game is a noisy signaling game. Therefore, the corresponding equilibrium concept, Perfect Bayesian Equilibrium, specifies to that of a Noisy Signaling Equilibrium (NSE). The Noisy Signaling Equilibrium prescribes equilibrium strategies for the firm and consumers which are sequentially rational to the other players' equilibrium strategies at each of their information sets (their beliefs about the consumers' privacy concerns), and beliefs which are consistent with the equilibrium strategies, coming from Bayesian updating. We focus on the unique differentiable equilibrium, ignoring pooling equilibrium. In this equilibrium, the firm's second period prices are a linear function of their private information. This is because if consumers use linear inference rules (ordinary least squares) to compute mean expectations about z, then linear decision rules by the firm will be optimal.

The next section offers the Bayesian updating of beliefs.

4 Updating of beliefs

Given the above information, we first calculate several Bayesian updaties for future references. Because all random variables are normally distributed, the Bayesian updates are just regression equations. In Appendix A, we offer the complete details of such calculations. First, we have the consumer's updated random variable α_{i1} once z has been observed. Let γ_z be the relative precision of signal z, i.e., $\gamma_z = \frac{\sigma_x^2}{\sigma_z^2}$, and let γ_x the relative precision of the prior distribution of

α_{i1}. To focus on signaling issues and keep aside manipulative behavior by the firm, we assume that γ_z is common knowledge among consumers and the monopolist. This basic case is enough to offer the main signalling results. The case when γ_z is not common knowledge is analyzed in Sánchez and Urbano (2019).[4]

By normality and the parameters of the corresponding distributions,

$$E\{\alpha_{i1}|z\} = \gamma_z z + \gamma_x \overline{x}, \tag{5}$$

where

$$\gamma_z = \frac{\sigma_x^2}{\sigma_z^2}, \gamma_x = 1 - \gamma_z = \frac{\sigma_z^2 - \sigma_x}{\sigma_z^2}. \tag{6}$$

In period 2, the consumers' posterior distribution of their privacy concerns comes from the information obtained through their purchases in period 1 and their updating of privacy concerns in period 1, α_{i1}. That is, from their previous experience in period 1 and their inference made on z from the monopolist's second period price. Thus, it is a linear combination of three relevant variables: the private's signal of the monopolist, z, the consumers' previous experience in period 1 on privacy issues, α_{i1}, and the average privacy in the population, $\overline{x}$, weighted by their relative precision, δ_z, δ_α and δ_x, respectively. Particularly, δ_α is the relative precision of the previous experience in period 1, δ_z is the relative precision of signal z in period 2 and δ_x is the relative precision of the prior distribution of α_{i2}. Therefore, the Bayesian updating of privacy concerns in period 2, conditional on z and α_{i1}, is given by

$$E\{\alpha_{i2}|\alpha_{i1}, z\} = \overline{x}\delta_x + \alpha_{i1}\delta_\alpha + z\delta_z, \tag{7}$$

where

$$\delta_\alpha = \frac{\sigma_x^2\sigma_\varphi^2 + \sigma_\omega^2\sigma_z^2}{\sigma_\alpha^2\sigma_z^2 - \sigma_x^4},$$

$$\delta_z = \frac{\sigma_x^2\sigma_v^2}{\sigma_\alpha^2\sigma_z^2 - \sigma_x^4},$$

and

$$\delta_x = 1 - \delta_\alpha - \delta_z.$$

In these equations, $\sigma_\alpha^2 = \sigma_x^2 + \sigma_\omega^2 + \sigma_\vartheta^2$ and $\sigma_z^2 = \sigma_x^2 + \sigma_\varphi^2$. Therefore,

$$\delta_z = \gamma_z (1 - \delta_\alpha), \tag{8}$$

[4] In Sánchez and Urbano (2019) the focus is on the manipulation of the market information by the monopolist, addressing two scenarios: direct investment in security to improve consumers' experiences and investment in the signal precision of z.

$$\delta_x = (1 - \gamma_z)(1 - \delta_\alpha). \tag{9}$$

Equations (8) and (9) show that updated beliefs depend on two key parameters which are δ_α and γ_z. Parameter δ_α is how much weight consumers put in their experience on privacy concerns from the online channel. Parameter γ_z is the relative precision of the monopolist's private information i.e., the relative precision of z. Note that an improvement in the precision of γ_z, means a decrease in the precision of γ_x, i.e., the relative precision of the true average privacy concern.

Before proceeding with the analysis under the different scenarios of equilibrium prices, it is interesting to think about the nature of equilibrium prices in period 2. At the beginning of period 2, both consumers and the firm have new information. Each consumer i remembers his first period experience, which yielded an observation of α_{it}, and the firm has an observation $\hat{z}$ of z. A high value of $\hat{z}$ indicates a high $\overline{x}$, which in turn indicates that consumer $i's$ expectation of α_{it}, is likely to be high. Thus, the firm concludes from a high $\hat{z}$ that second period demand is likely to be low (the difference between the willingness-to-pay for the product and privacy concerns), which points to a low maximizing price in period 2. Therefore, the firm has an incentive to learn the average privacy concerns in the online channel because her expected price and profits in period 2 depend on it. Consumers understand these incentives of the firm, and therefore, infer some information about the firm's observation of z from the price. Information about z is useful to consumers since it provides an independent signal of the true value of $\tilde{x}$, a component of their utility.

However, since each consumer's utility experience with the good is idiosyncratic, he will continue to use his personal information α_{it}, in making privacy concerns inferences. We conclude below that equilibrium does in fact posses these features; however, the presence of idiosyncratic signals to the consumers is crucial.

Next, two main scenarios or strategies are analyzed, where the monopolist can choose either to practice price discrimination between the brick and the click channel, or to set up a uniform price in both channels. First, we analyze the uniform price strategy and then price discrimination between channels, taking into account that the proportion of channels, λ, is exogenous.

5 Uniform pricing

In this section, we analyze how the monopolist sets up a uniform price in period $t = 1, 2$ given the set of information available for both the monopolist and consumers.

In period 1, consumers have expected demand given their set of information in $t = 1$. As above specified, no additional information for both the monopolist and consumers has been yet generated, and consumers' expected demand and firm's expected profits consists of simple expectations. Let p_1^u be the uniform price expected by consumers in period 1, and q_{iB1} and q_{iC1} the demands in period 1 of the brick and the on-line channels, respectively. Then, as usual, the demand in the brick channel is $q_{iB1}^u = \theta_i - p_1^u$, and the expected demand of consumer i in the online channel, conditional to her information set, $q_{iC1}^u = \theta_i - \overline{x} - p_1^u$.

The monopolist does not have any additional information neither, therefore, letting Π_1^u be the two-channels profits in period 1, then her expected profits, conditional to her information set are,

$$E\left[\Pi_1^u | \Omega_{m1}\left(\alpha_1\right)\right] = \lambda\left(\theta_i - p_1^u\right) p_1^u + \left(1-\lambda\right)\left(\theta_i - \overline{x} - p_1^u\right) p_1^u \left(1+r\right). \tag{10}$$

Similarly, let q_{iB2}^u and q_{iC2}^u be the expected demands in period 2 of the brick and the on-line channels respectively, and p_2^u is the uniform price expected by consumers in period 2. Then, the demand for the brick channel in period 2 is $q_{iB2}^u = \theta_i - p_2^u$, and $q_{iC2} = \theta_i - E\left\{E\left\{\alpha_{i2}|\alpha_{i1}, z\right\}|\alpha_{i1}, p_2^u\right\} - p_2^u$ is the expected demand in the online channel, conditional on his information set, at the beginning of period 2, which, given the updated beliefs of $E\left\{\alpha_{i2}|\alpha_{i1}, z\right\}$ (see equation (7) above) specifies to,

$$q_{iC2}^u = \theta_i - E\left\{\overline{x}\delta_x + \alpha_{i1}\delta_\alpha + z\delta_z | \alpha_{i1}, p_2^u\right\} - p_2^u.$$

The expected demand curve of the monopolist in period 2 is again the sum of the two channels expected demands. After period 1, she gets some new information about the consumers' average privacy and uses this information to set the second period's price. As in period 1, λ is the proportion of consumers buying in the brick channel and, r represents the extra benefits of sales of data. Letting Π_2^u be the two-channels profits in period 2 and, q_2^u period 2 demand, then, the expected demand faced by the firm and the monopolist's second-period expected profits are, respectively,

$$E\left[q_2^u | \Omega_{m2}\left(z, p_2\left(z\right)\right)\right] = \lambda q_{i2B}^u + \left(1-\lambda\right) E\left[q_{i2C}^u | z, p_2^u\right],$$

that specifies to

$$E\left[q_2^u|\Omega_{m2}\left(z,p_2\left(z\right)\right)\right]=\lambda\left(\theta_i-p_2^u\right)+\left(1-\lambda\right)\left(\theta_i-E\left\{\overline{x}\delta_x+\alpha_{i1}\delta_\alpha+z\delta_z|z,p_2^u\right\}-p_2^u\right), \quad (11)$$

and,

$$E\left[\Pi_2^u|\Omega_{m2}\left(z,p_2\left(z\right)\right)\right]=\lambda\left(\theta_i-p_2^u\right)p_2^u+\left(1-\lambda\right)\left(\theta_i-E\left\{\overline{x}\delta_x+\alpha_{i1}\delta_\alpha+z\delta_z|z,p_2^u\right\}-p_2^u\right)p_2^u(1+r). \quad (12)$$

5.1 Equilibrium

Once specified the consumers' expected demands and the monopolist's expected profits in $t = 1, 2$, we look for the market equilibrium. As said, the equilibrium concept is that of Noisy Signaling Equilibrium (NSE) and consists of: the monopolist's pricing strategy at each period, given her information sets Ω_{mt} in $t = 1, 2$, the consumers' expected demands coming from their utility maximization, given their sets of information Ω_{it} in $t = 1, 2$, and beliefs of both consumers and the monopolist, coming from Bayesian updating and consistent with the firm and the consumers' equilibrium strategies.

The first step to compute the NSE consists of exactly specifying what consumer i believes when he decides to purchase the product on the online channel at any possible information set, Ω_{i2}. A consumer that purchases the product on the brick channel knows exactly his utility, and thus he has no privacy concerns. However, a consumers information set at the beginning of period 2, of those buying on the online channel, consist of their own experience on α_{i1} plus the commonly observed p_2^u which possibly indicates the firm's observation of the monopolist's private signal, z.

Suppose that consumers make inferences on z from p_2^u according to the linear rule $z = a+bp_2^u$. Then, the online channel second period expected demand is,

$$q_{iC2}^u=\theta_i-\left\{\overline{x}\delta_x+\alpha_{i1}\delta_\alpha+\left(a+bp_2^u\right)\delta_z\right\}-p_2^u. \quad (13)$$

In period 2, the demand curve perceived by the monopolist is the expectation over the representatives consumer's demand conditional of her own information, Ω_{m2}, consisting of the observation $\hat{z}$, of z,[5] consumers' inference rule and second period price. Hence

$$E\left[q_2^u|\Omega_{m2}\left(z,p_2\left(\hat{z}\right)\right)\right]=\lambda\left(\theta_i-p_2^u\right)+\left(1-\lambda\right)\left(\theta_i-\left(\overline{x}\gamma_x+\delta_z\left(a+bp_2^u\right)+\hat{z}\delta_\alpha\gamma_z\right)-p_2^u\right). \quad (14)$$

[5] Given the assumption of normality of the random variables in the model, the obverved value of the signal, $\hat{z}$, can be either a positive or a negative value. We restrict our attention to the cases where $\hat{z} > 0$ in order to have positive prices.

Finally, the monopolist's second-period expected profits, taking into account the extra profits form data sales are,

$$E\left[\Pi_2^u|\Omega_{m2}\left(z,p_2\left(\hat{z}\right)\right)\right]=\lambda\left(\theta_i-p_2^u\right)p_2^u$$
$$+\left(\left(1-\lambda\right)\left(\theta_i-\left(\overline{x}\gamma_x+\delta_z\left(a+bp_2^u\right)+\hat{z}\delta_\alpha\gamma_z\right)-p_2^u\right)\right)p_2^u(1+r). \quad (15)$$

As can be seen from (14), the expected demand in period 2 shows that an increase in price has two distinct effects on demand. Reordering terms, we get that

$$E\left[q_2^u|\Omega_{m2}\left(z,p_2\left(\hat{z}\right)\right)\right]=\theta_i-\left(1-\lambda\right)\left(\overline{x}\gamma_x+\delta_z a+\hat{z}\delta_\alpha\gamma_z\right)+p_2^u\left(-1-b(1-\lambda)\delta_z\right).$$

In the above expression, the term (-1) represents the direct effect that the price has on expected demand in period 2. The indirect effect, which is the term $(-b(1-\lambda)\delta_z)$, depends on its sign.[6] We will see below that "b" is negative (see (20) below) and δ_z positive (see (8)), so that this term is positive. This means that an increase in prices translates to a consumer's higher inference value of his level of privacy concerns, α_{i2}, leading to a decrease in demand. If the value of $(-b(1-\lambda)\delta_z)$ is very high, i.e., if each consumer puts a lot of weight on p_2^u in drawing inferences about z, and places large weight on z when estimating his α_{i2} (see equation (7)) then, the demand curve will become flatter.

The important point to keep in mind is that an increase in prices will not reduce demand by as much as it would be in the absence of signaling. In other words, the fact than consumers draw inferences about privacy concerns from the price makes market demand less elastic at any particular quantity. [7]

Let the tuple (p_1^{u*},p_2^{u*}) be the equilibrium prices, i.e., given $E\left[\Pi_1^u|\Omega_{m1}\left(\alpha_1\right)\right]$ in $t=1$ and $E\left[\Pi_2^u|\Omega_{m2}\left(z,p_2\left(\hat{z}\right)\right)\right]$ in $t=2$, then

$$p_1^{u*}=\quad arg\quad \underset{p_1^u}{max}\quad\left\{\lambda\left(\theta_i-p_1^u\right)p_1^u+\left(1-\lambda\right)\left(\theta_i-\overline{x}-p_1^u\right)p_1^u\left(1+r\right)\right\}, \quad (16)$$

$$p_2^{u*}=arg\quad\max_{p_2^u}\left\{\lambda\left(\theta_i-p_2^u\right)p_2^u+\left(1-\lambda\right)\left(\theta_i-\left(\overline{x}\gamma_x+\delta_z\left(a+bp_2^u\right)+\hat{z}\delta_\alpha\gamma_z\right)-p_2^u\right)p_2^u(1+r)\right\}. \quad (17)$$

The first order conditions give the optimal price in period 2 yields,

$$p_2^{u*}=\frac{\theta_i(1+(1-\lambda)r)-(1-\lambda)(1+r)(\overline{x}\gamma_x+\hat{z}\delta_\alpha\gamma_z+a\delta_z)}{2\left(1+b(1-\lambda)(1+r)\delta_z+(1-\lambda)r\right)}. \quad (18)$$

[6]It is required that $\delta_z b(1-\lambda)<1$, thus the monopoly price always exists and the second-period pricing problem is always well defined.

[7]It is important to point out that our model imposes uncertainty in the intercept of the demand and not in its slope. Notice that some uncertainty in the slope would give consumers incentives to increase their purchases in order to increase their information (learning by experimentation). We leave experimentation issues aside.

We are searching for an equilibrium in which the representative consumer's inference rule is correct. Consumers are correct in believing that $\hat{z}$, the observation of z by the firm, is equal to $\hat{z} = a + bp_2^u$; then, the consumers' second period expected price p_2^u must satisfy that $p_2^u = (\hat{z} - a)/b$. At equilibrium, both consumers and the monopoly have the same linear rule for $\hat{z}$. Hence, prices have to be the same and this implies that the price set by the monopolist is also equal to $p_2^{u*} = (\hat{z} - a)/b$. Thus, matching term by term, a in (18) to $-a$ and terms with b to b, and solving for a and b, we obtain that in a linear equilibrium,

$$a = \frac{\theta_i(1 + r(1 - \lambda)) - \overline{x}\gamma_x(1 - \lambda)(1 + r)}{(1 - \lambda)(1 + r)\gamma_z}, \tag{19}$$

and

$$b = -\frac{2 + 2r(1 - \lambda)}{(1 - \lambda)(1 + r)\,(2 - \delta_\alpha)\,\gamma_z}. \tag{20}$$

Substituing a and b in (18), we get the expected price, and therefore, expected demand and profits in period 2. Proposition 1 characterizes the linear noisy signaling equilibrium.[8]

Proposition 1 *Suppose that the consumer proportion between channels is given by $\lambda \in (0, 1)$, the firm's mark-up is $r \in (0, 1)$ and inference rules are linear, then there exists a linear noisy signaling equilibrium with uniform pricing. In equilibrium,*

1. *The firm sets the price in $t = 1$,*

$$p_1^{u*} = \frac{\theta_i}{2} - \frac{(1 - \lambda)(1 + r)}{2(1 + (1 - \lambda)r)}\overline{x}.$$

and the expected demand in equilibrium is

$$q_1^{u*} = \frac{\theta_i}{2} + \frac{(1 - \lambda)(2\lambda r - (1 + r)}{2(1 + (1 - \lambda)r)}\overline{x}.$$

2. *In period 2, the second period price is*

$$p_2^{u*} = (2 - \delta_\alpha)(\frac{\theta_i}{2} - \frac{(1 - \lambda)(1 + r)}{2(1 + (1 - \lambda)r)}(\overline{x}\gamma_x + \hat{z}\gamma_z)), \tag{21}$$

and the expected demand in period 2 is

$$q_2^{u*} = \delta_\alpha\frac{\theta_i}{2} + \frac{(1 - \lambda)(2r\lambda - \delta_\alpha(1 + r))}{2(1 + (1 - \lambda)r)}(\overline{x}\gamma_x + \hat{z}\gamma_z)). \tag{22}$$

[8] Obviously, the second order conditions holds in each period. In period 1, the second order condition is $-2\lambda - 2(1 - \lambda)(1 + r) < 0$. Furthermore, in period 2, the second order condition is $2(1 - \lambda)(1 + r)\,(-b\delta_z - 1)$, and plugging "b" specified in (19) in the second order condition, we get $-\frac{2\delta_\alpha(1+r(1-\lambda))}{2-\delta_\alpha} < 0$.

Furthermore, this is the unique equilibrium where consumers' inferences about z are a differentiable and an invertible function of p_2^{u*}.

Proof. See Appendix B. ■

The second period price is indeed a linear function of signal $\hat{z}$. Furthermore, signaling might distorts prices upward in comparison to the scenario in which z is common knowledge to both consumers and the monopolist. To study this distortion, let p_2^z be the price in equilibrium in period 2 in which z is common knowledge i.e., both consumers and the monopolist receive information from signal z. Then, for any observation $\hat{z}$ of z, the equilibrium price is,

$$p_2^z = \frac{\theta_i}{2} - \frac{(1-\lambda)(1+r)}{2(1+(1-\lambda)r)}(\overline{x}\gamma_x + \hat{z}\gamma_z).$$

Clearly, $p_2^{u*} - p_2^z > 0$, and then, signaling always distorts prices upward compared to the scenario when z is common knowledge.

Additionally, recall that the consumers' learning process depends on two key parameters δ_α and γ_z, representing how much weight consumers put in their experience on privacy concerns from the online channel and the relative precision of the monopolist's private information, respectively. Let us analyze some interesting limiting cases.

If the precision of the consumers' experience δ_α tends to zero (this could be the case if $\gamma_z = 1$ and hence $\gamma_x = 0$ (see equation (6)) then, the second period expected price will be high (see equation (21)) and the monopolist will only sell to the brick channel (see equation (22)).[9] This limiting case corresponds to the case where the private information of the monopolist is a sufficient statistic for the consumers estimates of their private concerns. Then, every monopolist's type will have an incentive to set a high price with the purpose of signaling consumers that the average market privacy is low enough. Anticipating the monopolist's incentives, consumers will not trust her and then the only equilibrium demand of the click channel is zero. Therefore, in order that price is able to signal consumers the average market privacy it is necessary that the monopolist's private information not be a sufficient statistic. Some idiosyncratic information learned by consumers is important.

On the other hand, as consumers' private information becomes a sufficient statistic for the market privacy concerns, i.e., δ_α tends to one, then, the second period price and quantity tend

[9]When $\delta_\alpha = 0$,$\gamma_z = 1$ and$\gamma_x = 0$, then $P_2^{u*} = \theta_i - \frac{(1-\lambda)(1+r)\hat{z}}{(1+(1-\lambda)r)}$.

to the equilibrium monopoly price and quantity without the signaling of $\overline{x}$ by the firm.[10] In this limiting case, the monopolist has nothing to signal to consumers, because her information is useless for them.

For intermediate cases, we find that as consumers give more importance to their first period experience, the lower is period 2's expected price.[11] Also, any attempt by the monopolist to increase the relative precision γ_z of z, will only translate[12] to a price increase if $\overline{x} \geq \hat{z}$, i.e., if the true average privacy in the market is higher than the monopolist's private observation. This is so because after the observation $\hat{z}$, the higher the precision of z the more convinced is the monopolist that the inferred x is smaller than the true one $\overline{x}$, and then she will set a higher price. On the contrary, if $\hat{z} \geq \overline{x}$, then any increase of the precision of z will reafirm her beliefs that the inferred x is higher than the true one and, hence, she will set a lower price.

In addition, it is interesting to analyze how signaling and the uncertainty over consumers' privacy might distorts equilibrium prices either upward or downward in comparison to the equilibrium price in the scenario where the monopolist gets to know the true market average privacy concerns. To study this distortion, let p_2^F be the price under this monopolist's "noiseless information". In other words, the case in which the observation of the signal reveals the true population average privacy concerns without any noise. The equilibrium price is,

$$p_2^F = \frac{\theta_i}{2} - \frac{(1-\lambda)(1+r)}{2(1+(1-\lambda)r)}\overline{x}.$$

A sufficient condition[13] in order the difference $p_2^u - p_2^F$ is positive is that $\overline{x} \geq \hat{z}$. Thus, signaling distorts prices upward with respect to the "noiseless scenario" as long as the observed value of z is under the true $\overline{x}$. The monopolist believes that the market average privacy concerns are lower than they really are, and this makes her increase the price in period 2 with respect to the one under the true $\overline{x}$. Only when the realize value of z is sufficientely higher than $\overline{x}$, and δ_α is high enough the case is reverse ($p_2^u \leq p_2^F$), i.e., signaling does distort prices downward with respect the noiseless information monopoly price in period 2.

Corollary 1 *$p_2^{u*} > p_2^z$. Furthermore, $p_2^{u*} > p_2^F$ whenever $\overline{x} > \hat{z}$.*

[10]When $\delta_\alpha = 1$, since by (6), $\gamma_x = (1-\gamma_z)$, then $P_2^{u*} = \frac{\theta_i}{2} - \frac{(1-\lambda)(1+r)(\overline{x}+(\hat{z}-\overline{x})\gamma_z)}{2(1+(1-\lambda)r)}$, and $q_2^{u*} = \frac{\theta_i}{2} + \frac{((1-\lambda)(2\lambda r-(1+r))(\overline{x}+(\hat{z}-\overline{x})\gamma_z)}{2(1+(1-\lambda)r)}$, that for γ_z sufficiently small tends to P_1^{u*} and q_1^{u*}, the equilibrium price and quantity, respectively, when consumers do not take into account any signaling by the monopolist.

[11]This partial derivative of the expected price in period 2 is $\frac{\partial p_2^u}{\partial \delta_\alpha} = -\frac{(\theta_i(1+r(1-\lambda))-(1-\lambda)(1+r)(\overline{x}\gamma_x+\hat{z}\gamma_z))}{2(1+(1-\lambda)r)} < 0$.

[12]From (21), and since by (6), $\gamma_x = (1-\gamma_z)$, then $\frac{\partial p_2^u}{\partial \gamma_z} = \frac{(2-\delta_\alpha)(1-\lambda)(1+r)(\bar{x}-\hat{z}))}{2(1+(1-\lambda)r)} \gtreqless 0$ as $\overline{x} \gtreqless \hat{z}$.

[13]In particular, $p_2^u - p_2^F = (1-\delta_\alpha)(\frac{\theta_i}{2} - \frac{(1-\lambda)(1+r)}{2(1+(1-\lambda)r)}\overline{x}) - \frac{(2-\delta_\alpha)(1-\lambda)(1+r)}{2(1+(1-\lambda)r)}(\hat{z}-\overline{x})\gamma_z$.

6 Price discrimination

Our benchmark studies the simplest strategy that the monopolist can decide about her second-period price. However, the firm may choose to practice price discrimination between the two channels in order to extract the maximum willingness to pay.

Let the superscript "d" denote price discrimination. Then, let p_{1B}^d and p_{2B}^d be the prices of the brick channel in period 1 and 2, and let p_{1C}^d and p_{2C}^d be the prices of the online channel in period 1 and 2, respectively. In period 1, as in uniform pricing, consumers have expected demands given their set of information Ω_1 and the channel chosen for purchasing. As above, the demand for the brick channel is $q_{iB1} = \theta_i - p_{1B}^d$, and the expected demand for the online channel is: $E\left[q_{iC1}|\Omega_{i1}\left(\alpha_{i1}\right)\right] = \theta_i - \overline{x} - p_{1C}^d$.

Then, the monopolist expected profits in period 1 are,

$$E\left[\Pi_1^d|\Omega_{m1}\left(\alpha_1\right)\right] = \lambda\left(\theta_i - p_{1B}^d\right)p_{1B}^d + (1-\lambda)\left(\theta_i - \overline{x} - p_{1C}^d\right)p_{1C}^d\left(1+r\right).$$

In period 2, consumers' set of information changes to Ω_{i2}, and consumers update their beliefs about their privacy concerns -as long as they purchased through the online channel-. Now, the monopolist's decision is to set period 2 prices for the two channels, given her private information.

At equilibrium, the consumers' beliefs updating implies that period 2 consumers' expected demands are, $q_{iB2}^d = \theta_i - p_{2B}^d$ for the brick channel, and $q_{iC2}^d = \theta_i - E\left\{E\left\{\alpha_{i2}|\alpha_{i1}, z\right\}|\alpha_{i1}, p_{2C}^d\right\} - p_{2C}^d$, for the click one, where by (7) the second term in the right hand specifies to,

$$q_{iC2}^d = \theta_i - E\left\{\overline{x}\delta_x + \alpha_{i1}\delta_\alpha + z\delta_z|\alpha_{i1}, p_{2C}^d\right\} - p_{2C}^d,$$

and, by the inference linear rule, it translates to,

$$q_{iC2}^d = \theta_i - \left\{\overline{x}\delta_x + \alpha_{i1}\delta_\alpha + \left(a + bp_{2C}^d\right)\delta_z\right\} - p_{2C}^d.$$

The demand curve perceived by the monopolist is the sum of both demands, brick and click. After period 1, the monopolist gets some new information about the average privacy concerns, and she uses this information to set prices in both channels.

$$E\left[q_{i2}^d|\Omega_{m2}\left(z, p_{2C}^d\left(z\right)\right)\right] = \lambda q_{i2B}^d + (1-\lambda)E\left[q_{i2C}^d|z, p_{2C}^d\right].$$

that specifies to

$$E\left[q_{i2}^d|\Omega_{m2}\left(z, p_{2C}^d\left(z\right)\right)\right] = \lambda\left(\theta_i - p_{2B}^d\right) + (1-\lambda)\left(\theta_i - E\left\{\overline{x}\delta_x + \alpha_{i1}\delta_\alpha + z\delta_z|z, p_{2C}^d\right\} - p_{2C}^d\right).$$

Thus, the monopolist's second-period expected profits, taking into account the extra benefits of data sales are,

$$E\left[\Pi_2^d|\Omega_{m2}\left(z,p_2^d\left(\hat{z}\right)\right)\right]=\lambda(\theta_i-p_{2B}^d)p_{2B}^d$$
$$+(1-\lambda)(\theta_i-(\overline{x}\gamma_x+\delta_z\left(a+bp_{2C}^d\right)+\hat{z}\delta_\alpha\gamma_z))-p_{2C}^d)p_{2C}^d(1+r). \quad (23)$$

Once we have specified the expected demands and the expected benefits in $t=1,2$, let the tuple $(p_{1B}^{d*},p_{1C}^{d*})$ be the equilibrium prices for period 1 and $(p_{2B}^{d*},p_{2C}^{d*})$ those for period 2, i.e.,

$$p_1^{d*}=\underset{p_{1B}^d,p_{1C}^d}{arg\quad max}\quad\left\{\lambda\left(\theta_i-p_{1B}^d\right)p_{1B}^d+(1-\lambda)\left(\theta_i-\overline{x}-p_{1C}^d\right)p_{1C}^d\left(1+r\right)\right\} \quad (24)$$

and

$$p_2^{d*}=\underset{p_{2B}^d,p_{2C}^d}{arg\quad max}\quad E\left[\Pi_2^d|\Omega_{m2}\left(z,p_2^d\left(\hat{z}\right)\right)\right] \quad (25)$$

In particular, given the expected demand perceived by the monopolist, its optimal price in period 2 in the click channel is

$$p_{2C}^{d*}=\frac{\theta_i-a\delta_z-(\overline{x}\gamma_x+\hat{z}\delta_\alpha\gamma_z)}{2(1+b\delta_z)}. \quad (26)$$

Furthermore, in equilibrium, the representative consumer's inference rule is correct. Thus, consumers are correct believing that the observation $\hat{z}$ by the firm equals $\hat{z}=a+bp_{2C}^d$; then, the consumers' second period expected price in the click channel satisfies that $p_{2C}^d=(\hat{z}-a)/b$, and hence, the monopolist's second period price for the online market is also equal to $p_{2C}^{d*}=(\hat{z}-a)/b$. By (26), in a linear equilibrium:

$$a=\frac{\theta_i-\overline{x}\gamma_x}{\gamma_z}, \quad (27)$$

and

$$b=\frac{2}{\left(\delta_a-2\right)\gamma_z}. \quad (28)$$

Substituing "a" and "b" in (26), we get the expected price, and therefore, expected demand and profits in period 2 in the online channel.

Proposition 2 characterizes the linear noisy signaling equilibrium.[14]

Proposition 2 *Suppose that market-channel proportions are given by $\lambda\in(0,1)$, the firm's mark-up is $r\in(0,1)$, and the inference rules are linear. Then, there exists a linear noisy signaling equilibrium with price discrimination. In equilibrium,*

[14] The second order conditions are again trivially satisfied. For the brick channel in period 1 and 2, the second order condition is $-2\lambda<0$. In the online channel, the second order condition is $-2(1-\lambda)(1+r)<0$ in period 1, and $2(1-\lambda)(1+r)(-1+b\delta_z)<0$ in period 2. Plugging "b", as in (28), in the second order conditions for the click channel results in a negative sign, i.e., $-\frac{2(1-\lambda)(1+r)\delta_\alpha}{2-\delta_\alpha}<0$.

1. *In period 1, the equilibrium quantities and prices for the brick channel are*

$$p_{1B}^{d*} = \frac{\theta_i}{2}, \qquad q_{1B}^{d*} = \lambda \frac{\theta_i}{2},$$

and for the click channel,

$$p_{1C}^{d*} = \frac{1}{2}(\theta_i - \overline{x}), \qquad q_{1C}^{d*} = \frac{1}{2}(1-\lambda)(\theta_i - \overline{x}).$$

2. *In period 2, the equilibrium quantities and prices for the brick channel are*

$$p_{2B}^{d*} = \frac{\theta_i}{2}, \qquad q_{2B}^{d*} = \lambda \frac{\theta_i}{2} \tag{29}$$

and in the click channel,

$$p_{2C}^{d*} = \frac{1}{2}(2 - \delta_\alpha)(\theta_i - (\overline{x}\gamma_x + \hat{z}\gamma_z)), \tag{30}$$

with expected quantity,

$$q_{2C}^{d*} = \frac{1}{2}(1-\lambda)\delta_\alpha (\theta_i - (\overline{x}\gamma_x + \hat{z}\gamma_z)).$$

Furthermore, the second period equilibrium price in the click channel p_{2C}^{d*} is the unique equilibrium where consumers' inferences on $\hat{z}$ are a differentiable and an invertible function of p_{2C}^{d*}.

Proof. See Appendix B. ■

The distortion on the second period online channel price because of market signaling follows the same pattern of the one with uniform princing. As expected, the second period price in the click channel is a linear function of $\hat{z}$ and, under price discrimination, the learning process only affects to the second-period price. Again, the monopolist will distort second period prices upward in the click channel in comparison to the scenario in which z is common knowledge to both consumers and the monopolist. To study this distortion, let p_{2C}^{z} be the price in equilibrium in period 2 for the click channel in which z is common knowledge. At equilibrium,

$$p_{2C}^{z} = \frac{1}{2}(\theta_i - (\overline{x}\gamma_x + \hat{z}\gamma_z)),$$

and trivially,

$$p_{2C}^{d} - p_{2C}^{z} = \frac{1}{2}(1 - \delta_\alpha)(\theta_i - (\overline{x}\gamma_x + \hat{z}\gamma_z)) > 0.$$

Similar analysis and intuition that those in the uniform case can be translated to the limiting case where the private information of the monopolist is a sufficient statistic for the consumers estimates of their private concerns in the click channel, i.e., when δ_α tends to zero. By equation (30), and in an attempt to manipulate consumers' privacy concerns, the monopolist will set a high second period price on the online market[15]. However, consumers will not be deceived and the monopolist will only sell to the brick channel, i.e., $q_{2c}^{d*} = 0$. On the contrary, as consumers' private information becomes a sufficient statistic for the market privacy concerns, i.e., when δ_α tends to one then, the online market second period price and quantity will tend to those price and quantity without market signaling by the monopoly, respectively, i.e., $P_{2C}^{d*} = \frac{1}{2}(\theta_i - \overline{x})$ and $q_{2C}^{d*} = \frac{1}{2}(1-\lambda)(\theta_i - \overline{x})$. Finally, for intermediate cases, the period 2's expected price in the click channel is lower the higher is the importance that consumers give to their first period purchasing experience.

Finally, as in uniform pricing, let P_{2C}^{F} be the price set by the monopolist when the observation of z reveals her the true $\overline{x}$, then a sufficient condition[16] for $p_{2C}^{d*} - P_{2C}^{F} > 0$ is that $\overline{x} \geq \hat{z}$. Again, signaling distorts prices upward as long as the observed value of z is under the true $\overline{x}$, and downward when the realized value of z is sufficientely higher than $\overline{x}$, and δ_α is high enough.

Note also that under a price discrimination, the second period prices in both markets do not depend on the mark-up earned by the monopolist, neither on parameter λ. The price in the brick channel does not change over the two periods.

Corollary 2 *$p_{2C}^{u*} > p_{2C}^{z}$. Furthermore, $p_{2C}^{u*} > p_{2C}^{F}$ whenever $\overline{x} > \hat{z}$.*

7 Comparison of pricing strategies

7.1 Equilibrium profits

Once we have computed the equilibrium prices and quantities of the two pricing strategies, we wish to know whether the channel-based price differentiation is an opportunity for the monopolist to increase expected profits. Thus, the following Proposition compares expected profits under the two pricing strategies.

Proposition 3 *The monopolist's expected profits in period 2 are higher under channel-based price discrimination. Furthermore, expected profits in the two-period game are also higher under channel-based price discrimination.*

[15] $p_{2C}^{d*} = (\theta_i - (\overline{x}\gamma_x + \hat{z}\gamma_z)$.

[16] $p_{2C}^{d*} - P_{2C}^{F} = \frac{1}{2}((1-\delta_\alpha)(\theta_i - \overline{x}) - (2-\delta_\alpha)(\hat{z} - \overline{x})\gamma_z)$.

Proof. See Appendix B. ■

Therefore, the monopolist will get higher expected profits in period 2, i.e., $E\Pi_2^{d*} - E\Pi_2^{u*} > 0$, if she discriminates over channels, and sets different second-period prices in our two-channel market with signals and learning. Similarly, let $E\Pi_a^{u*}$ and $E\Pi_a^{d*}$ be the aggregate profits in both periods under uniform price strategy and price discrimination, respectively, then $E\Pi_a^{d*} - E\Pi_a^{d*} > 0$.

Summing up, we get that under price discrimination not only expected profits are higher in period 2, when market signals play a crucial role, but also are aggregate expected profits. Therefore, these results are in line with the standard microeconomic result that a monopolist will get higher profits if she price discriminates between markets with different elasticities. Here, as a novelty, the existence of consumers' privacy concerns and market learning from (noisy) signals produce this result. Therefore, such consumers' privacy concerns should be taken into consideration when the monopolist designs her pricing strategy.

7.2 Are click channel prices consistently lower than those of brick channels?

In the previous section, the results show that engaging in a channel-based price differentiation increases the monopolist's profits. In a market with heterotegeneous tastes and different product valuations, companies may increase their profits by segmenting consumers and charging different prices, which allows them the extraction of additional consumer surplus.

Nevertheless, these findings might contradict existing empirical studies on price variability. For example, Cavallo (2017) finds that there is significant heterogeneity in pricing behaviors across retailers: those with nearly identical online and offline prices, those with stable online markups (either positive or negative), and those with different prices that are not consistently higher or lower online. Specifically, he claims that online and offline prices are identical about 72% of the time, that implies little within-retailer price dispersion. This result is much in line with the widespread idea of consistent prices across channels in order to maintain a strong brand and channel price integrity (Campbell and Campbell 2010). On the other hand, there are empirical papers that support the existence of price dispersion among channels. Cuellar and Brunamonti (2014) find price dispersion for a single item across retail channels. Additionally, and contrary to the common accepted possibility that online prices are more expensive than offline because of the possibility of tailored offers, Wolk and Ebling (2010) conclude that multi-channel retailers charge on average higher prices through the offline channel. There are several possible explanations that could shed light on this matter: price dispersion based on demographic self-selection and shopping intent (Cuellar and Brunamonti 2014), or the perceived risk in the online channel (Wolk and Ebling 2010). Cavallo (2017) also analyzes price dispersion based on IP addresses or browsing habits (very controversial causes), but surprisingly, he does not find any evidence to support these causes.

Given that there is not a general consensus on whether online prices are higher or lower than offline prices, and, what is more, whether a uniform price would be higher or lower than each of them, we seek to isolate the endogeneous variable which may explain the different equilibrium price orderings that may take place.

In this searching, our model is able to show that an important determinat of the different price orderings is the population-average privacy in the market, suggesting a possible explanation to the dispersion of prices between market channels. Indeed, there are several previous studies trying to determine the value of privacy in the market and showing that privacy concerns have the greatest impact on purchasing intents (Eastlick et al. 2006). Furthermore, a recent lab experiment, suggest that agents care a great deal about their privacy and what is more important, they assign a positive value to their privacy, and therefore, they experience a disutility from giving up privacy, disminishing their willingness to pay relative to the expected value, (Biener et al. 2020). Following this view, and given the growing importance that represents knowing the value of the population-average privacy for companies and policy makers, we try to see whether it may explain price orderings.

Note, that in our setting with signals and market learning the role of the market average privacy is nuanced by the importance that consumers place on their online channel previous experience. Nevertheless, and independently of the consumer experience, we find two thresholds values of the market average privacy that lead to changes in price orderings. Furthermore, the uniform price (setting an identical price in both channels) is not always the lowest price that can be set compared to those of channel price discrimination.

7.2.1 Market population-average privacy and price orderings

We seek to analyze how equilibrium prices under the two pricing strategies are related. We first consider price discrimination and analyze how the price of the brick channel relates to that of the click channel. That is, whether $p^d_{2B} - p^d_{2C} \gtreqless 0$.

Consider any realized $\hat{z}$ and fix the parameter of the model but $\overline{x}$, then there is a certain value of the market average privacy, $\overline{x}_1$, for which both prices coincide. Hence, for any average privacy lower than $\overline{x}_1$, we find that the price in the click channel is higher than the one in the brick one, meaning that when consumers do not have too many privacy concerns on average, the monopolist finds it profitable to set a higher price online. Notice, that when $\overline{x} \to 0$, prices do not converge to the same point because although the idyosincratic privacy is 0, the residual privacy still exists in the market i.e., this is because of the learning process and the previous experience that consumers have in period 1. On the other hand, for any other value of the market average privacy higher than $\overline{x}_1$, the result is reversed, which is consistent with the fact that if consumers value privacy a lot in average, then prices in the online market will have to be lower than those of the brick channel in order to incentive online purchases.

Calculating this threshold by equating (29) and (30), and leaving $\overline{x}$ alone yields,

$$\overline{x}_1 = \frac{\theta_i (1 - \delta_\alpha)}{(2 - \delta_\alpha) \gamma_x} - \frac{\hat{z}\gamma_z}{\gamma_x}.$$

As already mentioned, the weight that consumers put in their previous privacy experience will affect the value of $\overline{x}_1$. Namely, the higher is δ_α, the smaller is $\overline{x}_1$ and viceversa.

Next, we analyze how the equilibrium uniform price is regarding to equilibrium discriminatory prices. It is easy to check that the uniform price is always higher than the equilibrium price for the click channel i.e., $p_2^u - p_{2C}^d > 0$. The price difference is equal to,

$$p_2^u - p_{2C}^d = \frac{\lambda (2 - \delta_\alpha) (\overline{x}\gamma_x + \hat{z}\gamma_z)}{2(1 + (1 - \lambda) r)} > 0$$

which is positive. Surprisingly, this is not the case when the uniform price is compared with the equilibrium price of the brick channel. Comparing p_2^u and p_{2B}^d, we find a new threshold that makes a point of change in the order on prices.

This new threshold value is obtained by equating (21) and (29) and solving for $\overline{x}$. Let $\overline{x}_2$ denote such a threshold, also decreasing in δ_α,

$$\overline{x}_2 = \frac{\theta_i (1 + (1 - \lambda) r) (1 - \delta_\alpha)}{\gamma_x (2 - \delta_\alpha) (1 - \lambda)(1 + r)} - \frac{\hat{z}\gamma_z}{\gamma_x},$$

Comparing the two threshold, $\overline{x}_1$ and $\overline{x}_2$, we get

$$\bar{x}_2 - \bar{x}_1 = \frac{\theta_i \lambda (1 - \delta_\alpha)}{\gamma_x (2 - \delta_\alpha) (1 - \lambda)(1 + r)} > 0,$$

which is positive and decreasing in δ_α, meaning that the higher δ_α the wider the range of the market average privacy values for which both the uniform and brick channel prices are higher than the click channel price. This make sense in our context because the higher the role of consumers' privacy concerns the lower the price online that the monopolist sets.

The above price differences contradict the empirical findings in Cavallo (2017) who claims that channel prices are identical about 72% of the time. Nevertheless, our results are more in line with Cuellar and Brunamonti (2014), who find that retail channel is an effective means of price discrimination based on demographic self-selection and shopping intent.

The two thresholds values define three sections in the cartesian axes relating prices and market average privacy. To illustrate, figure 1 plots how prices orderings change as a function of the market average privacy, for some specific values of the variables and parameters of the model. Namely, figure 1 (a) plots the price orderings when the weight that consumers give to their previous experience with privacy is high, i.e., $\delta_\alpha = 0.9$, that corresponds with a low value in the private signal's precision of the monopolist, $\gamma_z = 0.1$. In the same way, figure 1 (b) represents price orderings for a value of the signal's precision $\delta_\alpha = 0.1$ and $\gamma_z = 0.9$. It is important to recall that high values of the weight of consumers' experience, δ_α, yield low values of $\overline{x}_1$ and $\overline{x}_2$ and also a small difference between them; conversely, low values of δ_α yield high values of $\overline{x}_1$ and $\overline{x}_2$ and a big difference between them. This fact means that the average privacy levels from which there is a change in the price orderings are greater as the weight δ_α decreases.

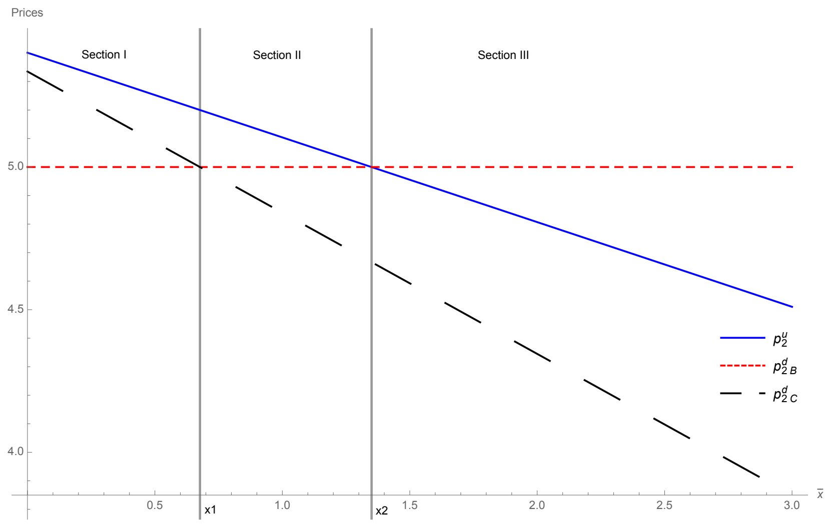
Clearly, for low values of the market average privacy $\overline{x}$, in particular for $\overline{x} < \overline{x}_1$, the monopolist will set a price in the online channel higher than the one in the brick channel; also the price under uniform pricing will be higher than the one in the brick channel, given that $\overline{x}_1 < \overline{x}_2$. The situation is reversed for high values of $\overline{x}$, specifically for $\overline{x} > \overline{x}_2$. Now, the monopolist will set a price in the click channel, lower than that in the brick one; the price under uniform pricing will also be low and, in particular, lower than that in the brick channel. Finally, for an intermediate market average privacy, $\overline{x}$, $\overline{x}_1 < \overline{x} < \overline{x}_2$, the uniform price is still higher than that of the brick channel and the price in the click channel is the smallest one.

To sum up, the above findings are stated in the following proposition:

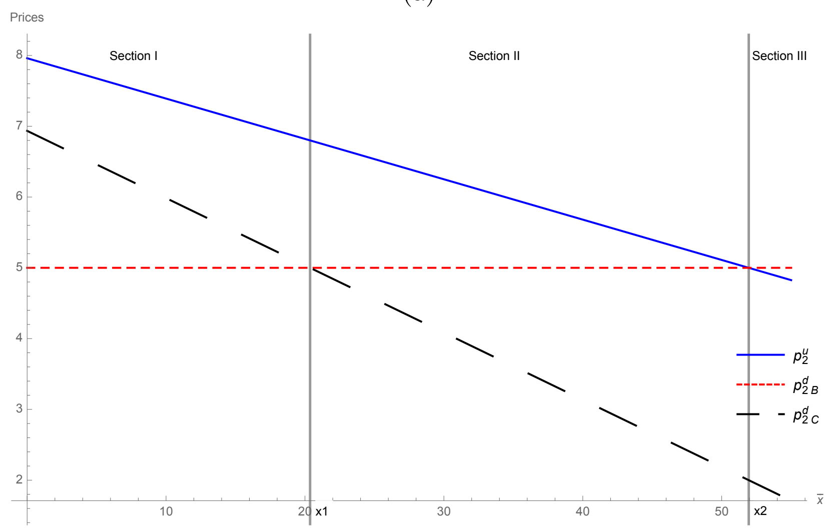
Proposition 4 *1. For small values of the market average privacy, $\overline{x} < \overline{x}_1$, the price ordering is $p_2^u > p_{2C}^d > p_{2B}^d$ (Section I in figure 1 (a, b).)*

2. For intermediate values of the average privacy in the market, $\overline{x}_1 < \overline{x} < \overline{x}_2$, the price ordering is $p_2^u > p_{2B}^d > p_{2C}^d$ (Section II in figure 1 (a, b).)

3. For high values of the market average privacy, $\overline{x}_2 < \overline{x}$, the price ordering is $p_{2B}^d > p_2^u > p_{2C}^d$ (Section III in figure 1 (a, b).)



(a)



(b)

Figure 1: In graph (a), we plot prices for the following parameter values: $\theta_i = 10$, $r = 0.5$, $\lambda = 0.5$, $\hat{z} = 3$, $\gamma_x = 0.9$, $\gamma_z = 0.1$ and $\delta_\alpha = 0.9$. In graph (b) $\theta_i = 10$, $r = 0.5$, $\lambda = 0.5$, $\hat{z} = 3$, $\gamma_x = 0.1$, $\gamma_z = 0.9$ and $\delta_\alpha = 0.1$

4. Both bounds $\overline{x}_1$ and $\overline{x}_2$ decrease as δ_α increases.

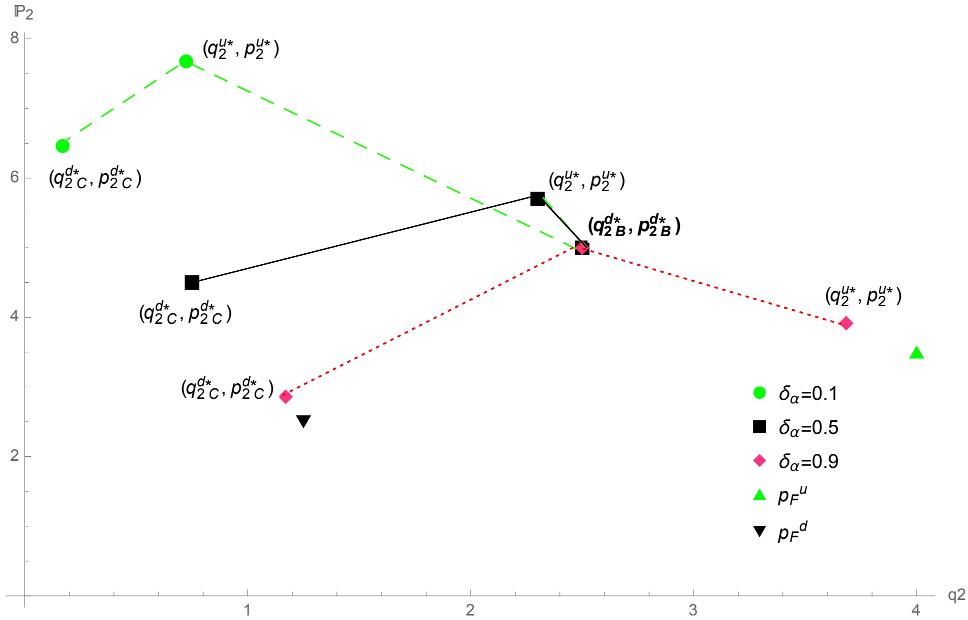
Note, however, that the monopolist does not know $\overline{x}$, but observes a realization of $z = \tilde{x} + \tilde{\varphi}$, where $\tilde{x} \sim N(\bar{x})$, and makes an inference on it. Thus, once $\hat{z}$ is observed, its value will determine her inference on $\overline{x}$, and hence the monopolist's price setting behavior. If the observed value $\hat{z}$ is big enough, then the monopolist's inference on the value of $\overline{x}$ will result in a high value of it. Therefore, her best response is to lower the online price with respect the not signaling or "noiseless" case ($\overline{x}$ known for sure) with lower level prices than the ones in the brick channel (section II and III of figure 1). Also de uniform price will go down with respect to the noisless case, and depending on the relative decrease of the online price that of the brick channel will be higher or lower than the former. On the contrary, if the observed value $\hat{z}$ is low enough, the monopolist's inference results in a low value for the average privacy concerns, the monopolist increases the price online with respect to the noiseless case, as in section I of figure 1. As above the uniform price will also increase. Therefore we may find a hugue price variability, among the different channel prices and the uniform one. Extending this analysis to an industry with several firms selling similar products, we may encounter price dispersion (Hopkins et al., 2006).

To conclude with, we find that in case of price discrimination beetween two sales channels, the online prices can be higher or lower than the offline prices depending on the average privacy concerns in the market. In addition, and contrary to what is suggested in the literature, online and offline prices can be smaller than those charged under uniform pricing in a dual-channel context. Thus, we find that there it might exist multichannel market price dispersion. Consumers' privacy concerns and market signaling are important elements that affect the monopolist's expected profits, and therefore, their optimal price decisions.

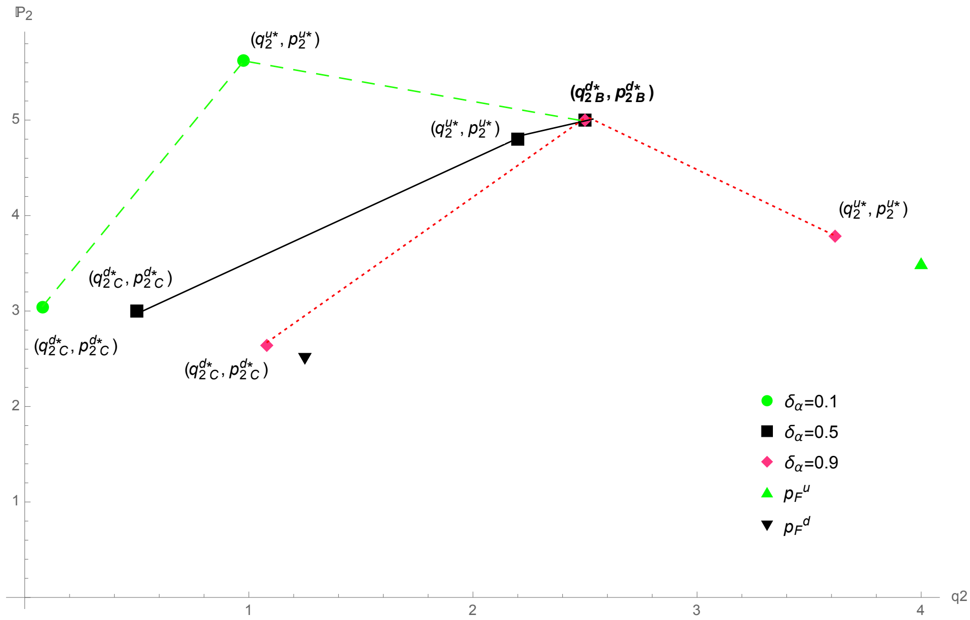
7.3 Equilibria, signaling and market learning

To better grasp a general picture of our findings we plot second period equilibrium points under uniform pricing and price discrimitation as a function of the signaling-learning parameters. On the one hand, the monopolist learns about $\overline{x}$ by observing signal z. On the other, consumers learn about their privacy concerns by updating α_{i1} and by observing second period prices. Therefore, we fix the value of all parameters of the model and let the precision of δ_α, take three values, for $\overline{x} > \hat{z}$ and $\overline{x} < \hat{z}$.

In figure 2, graph (a) refers to the situation where $\overline{x} > \hat{z}$, and graph (b) for the reversed case. In both graphs, the green dashed line joint the equilibrium points for $\delta_\alpha = 0.1$ i.e., when



(a)



(b)

Figure 2: Equilibrium points depending on the weight that consumers give to their previous experience, δ_α. The value of parameters are: $\theta_i = 10$, $r = 0.5$, $\lambda = 0.5$. In graph (a), we plot for $\overline{x} = 5$ and $\hat{z} = 3$, thus $\overline{x} > \hat{z}$. However, graph (b) presents the reversed case with $\overline{x} = 5$ and $\hat{z} = 7$.

consumers give a low weight to their previous privacy experience and hence, its information's precision is low. In the same way, the black line joint the equilibrium points for $\delta_\alpha = 0.5$ medium weight for the importance of privacy. Finally, the red dotted line joint the equilibrium points for $\delta_\alpha = 0.9$, almost the highest privacy experience concerns.

In graph (a), $\overline{x} > \hat{z}$, and for δ_α=0.1, the lower threshold of $\overline{x}$ is $\overline{x}_1 = 20.36$, and since $\overline{x} = 5$, then $\overline{x} < \overline{x}_1$ and, as expected (see Proposition 4, Section I in figure 1), $p_2^u > p_{2C}^d > p_{2B}^d$. For $\delta_\alpha = 0.5$, then $\overline{x}_1 = 3.66$, and $\overline{x}_2 = 8.11$, and thus, $\overline{x}_1 < \overline{x} < \overline{x}_2$ and therefore, $p_2^u > p_{2B}^d > p_{2C}^d$ (Proposition 4, Section II in figure 1). Finally, for $\delta_\alpha = 0.9$, $\overline{x}_1 = 0,676$, and $\overline{x}_2 = 1,350$, and thus, $\overline{x} > \overline{x}_2$, and consequently, $p_{2B}^d > p_2^u > p_{2C}^d$ (Proposition 4, Section III in figure 1). Note that as δ_α increases, then both $\overline{x}_1$ and $\overline{x}_2$ decrease.

Also note that both the second period uniform and click channel prices are higher than the corresponding $P_2^z(\delta_\alpha)'s$, (when z is common knowledge) and since $\overline{x} > \hat{z}$, then both prices are also higher than the "noiseless" case prices, where $\overline{x}$ is common knoledge, i.e. $P_2^F(\delta_\alpha)'s$, as stated in corollaries 1 and 2, respectively. Namely, under unifor pricing $P_2^z(\delta_\alpha = 0.1)$=4.04, $P_2^z(\delta_\alpha = 0.5)$=3.8 and $P_2^z(\delta_\alpha = 0.9)$=3.56, and $P_2^F = 3.5$. Similarly, under price discrimination $P_{2C}^z(\delta_\alpha = 0.1) = 3.4$, $P_{2C}^z(\delta_\alpha = 0.5) = 3$ and $P_2^z(\delta_\alpha = 0.9) = 2.6$ and $P_{2C}^F = 2.5$.

Now, in graph (b), $\overline{x} < \hat{z}$, and simple calculations show that $\overline{x}_2(\delta_\alpha = 0.1) = 15.0 > \overline{x} > \overline{x}_1(\delta_\alpha = 0.1)$, and then $p_2^u > p_{2B}^d > p_{2C}^d$. For both δ_α=0.5 and $\delta_\alpha = 0.9$, $\overline{x} > \overline{x}_2$ with $p_{2B}^d > p_2^u > p_{2C}^d$. Again, the both second period uniform and click channel prices are higher than the corresponding $P_2^z(\delta_\alpha)'s$, and both prices are also higher than the $P_2^F(\delta_\alpha)'s$.

Let us make some final comments about the equilibrium points in figure 2. The equilibrium price and quantity for the brick channel does not depend on δ_α, thus there is a unique equilibrium point for the brick channel, denoted by the subscript "2B", which is the full information price and quantity. In graph (a), the highest price is achieved when $\delta_\alpha = 0.1$ and under uniform pricing. However, the lowest price is achieved when $\delta_\alpha = 0.9$ in the click channel, which is consistent with the idea that a high importance of consumers' previous privacy experiences make the monopolist to set a lower price in the click channel under price discrimination. Moreover, it is easily seen that the presence of signals in the market distorts upwards prices with respect to the scenarios where z is common knowledge and where $\overline{x}$ (the noiseless case) is known.[17] Prices in the noiseless case are represented in the graph by the green triangle (under uniform price

[17]In the noiseless case and for $\hat{z} > \overline{x}$, it is needed that z is big enough in order that signaling distorts downwards equilibrium prices. This is not the case in the above simulation.

strategy) and black triangle (under price discrimination). Furthermore, in graph (b), prices follow the same path describe above but equilibrium prices in general decrease with $\hat{z} > \overline{x}$.

8 Main Highlights and Final Remarks

The paper presents a monopolist that operates in a dual-channel context, brick and click channels. She has to set prices to maximize her expected profits. In principle, consumers' demands are the same for the two channels but the one in the Internet is affected by consumers' privacy concerns, which are unknown for them. The monopolist receives a private signal with noise (noisy signal) about the market average privacy, which allows her to adjust the price in the online channel. The prices that the monopolist designs in the second period will serve as a signal to consumers about the use of their privacy, and this, together with their experience in the first period, will determine their demand. Thus, in a dynamic setting, consumers and the monopolist are learning from the signals in the market. Signaling may distort prices upwards with respect to those where z is common knowledge, and upwards or downwards with respect to those of the noiseless scenario. We find out that the monopolist gets higher expected profits under channel-based price discrimination. Furthermore, it does exist dispersion in the differentiation over channels and the uniform price, much in line with the literature. Nevertheless, this dispersion depends on the population-average privacy concerns in the market, and there is not a clear behaviour on which channel the price is higher or lower and if they are higher or lower than uniform prices. The existence of consumers' privacy can be understood as a possible explanation for multichannel market price dispersion and a key factor for the design of online prices.

Two questions deserve some attention. First, the fact that the monopolist's signal precision is common knowledge avoids manipulative behavior by the firm. The monopolist has incentives to signal lower levels of signal's precision in the market to make consumers believe that her precision is worse than it really is. By doing so, the monopolist increases consumers' trust in the online market, thereby increasing consumers' market demand. These results also show an interesting trade-off between the level of expected price and expected demand in period 2. Whereas expected price increases with the market signal, the effect is the opposite with respect to the expected demand. If the monopolist manipulates the market signal precision, consumers will pay more attention to their own experience and less to the market signal. This shift in attention increases expected second period demand and results in a lower expected second period price than the one in absence of manipulation. The optimal choice is the one that increases the monopolist's expected profits, so the demand effect dominates the price effect. According to the results of Sanchez and Urbano (2019), the monopolist has an incentive to create less confidence in the market signal (the public signal) and more in the consumers' individual experiences (the consumers' private signal).

On the other hand, an interesting question to address is how the heterogeneity in consumers' information affect the nature of equilibrium and the social welfare in this market. For example, suppose that $\rho \in (0, 1)$ is the proportion of consumers in the online channel who receive a signal about their privacy concerns at the end of the first period because they have purchased the product via online in period 1; they are "savvies" or experienced. Thus, $(1-\rho)$ is the proportion of consumers who have not received any signal about their privacy concerns because they did not purchase the product in period 1, and therefore, they are "non-savvies" or uninformed. It is not difficult to show that both the consumer surplus and the social surplus depend on the proportion of the more experienced consumers in the online channel. Indeed, the higher the level of consumers' information, the higher the social welfare and the consumer surplus attainable in this market.

Our preliminary results, point out that the presence of more experienced consumers on privacy concerns increase social welfare in the market. That suggest, in line with regulations about consumers' privacy in digital markets, that the higher the control that consumers have about their information, the higher the welfare that can be achieved in the marketplace. Furthermore, the monopolist's profits in period 2 are increasing in the proportion of informed consumers. Thus, social surplus is also increasing in the proportion of more informed agents, ρ. Therefore, the existence of heterogeneity in the consumers' information, in particular, uniformed consumers, harm the whole market in general, whereas homogeneously informed consumers guarantee gains in the market, and hence a higher social surplus.

We leave other relevant questions aside. For example, we assume fixed the proportion between channels. Having the proportions of channels depending on prices would give rise to interesting questions where prices and privacy concerns will lead the a consumers flow between channels. How much privacy are you willing to give up in order to get a better price? This can be a fascinating idea for future research.

Appendix A: Bayesian updating

Given that all random variables are normally distributed, Bayesian updates are just regression equations. First, we have the Bayesian updating of the random variable α_{i1} once the private signal, z, has been observed. To start with, we compute the expected value, the variance and the correlation of α_{i1} and z taking into consideration that these random variables are specified in (3) and (4):

1. The expected values of α_{i1} and z are $E\{\alpha_{i1}\} = E\{z\} = \overline{x}$.

2. The variance of α_{i1} and z are $Var(\alpha_{i1}) = \sigma_x^2 + \sigma_\omega^2 + \sigma_\upsilon^2$ and $Var(z) = \sigma_x^2 + \sigma_\varphi^2$. In order to simplify, we just call $Var(\alpha_{i1}) = \sigma_\alpha^2$ and $Var(z) = \sigma_z^2$

3. The correlation between the two variables is specified by the index ρ. Calculations gives that $\rho = \frac{\sigma_x^2}{\sigma_\alpha \sigma_z}$.

   Now, note that, following DeGroot (2005), the Bayesian updating of the mean with normal random variables when the variance is known, is

$$\mu' = \frac{\tau\mu + ns\overline{x}}{s + ns}, \tag{A.1}$$

   where μ and τ are the prior mean and precision, respectively, and s is the poterior precision given n observations of a random sample. In our scenario with correlated variables, the Bayesian updating translates to:

$$E\{\alpha_{i1}|z\} = E\{\alpha_{i1}\} + \rho\frac{\sigma_\alpha}{\sigma_z}(z - E\{z\}), \tag{A.2}$$

   with $\rho(\alpha_{it}, z) = \frac{Cov(\alpha_{it}, z)}{\sqrt{Var(\alpha_{i1})Var(z)}}$.

   Substituting in (A.2) the corresponding terms, we get the following expression:

$$E\{\alpha_{i1}|z\} = \overline{x}\left(1 - \frac{\sigma_x^2}{\sigma_z^2}\right) + z\left(\frac{\sigma_x^2}{\sigma_z^2}\right).$$

Letting γ_z be the relative precision of signal z, i.e., $\gamma_z = \frac{\sigma_x^2}{\sigma_z^2}$, and γ_x the relative precision of the prior distribution of α_{i1}, i.e, $\gamma_x = 1 - \gamma_z = \left(1 - \frac{\sigma_x^2}{\sigma_z^2}\right)$. Recall that we have assumed that γ_z is common knowledge among the firm and consumers.

$$E\{\alpha_{i1}|z\} = \gamma_z z + \gamma_x \overline{x}. \tag{A.3}$$

Clearly, the Bayesian updates of α_{i1} conditional to z is a linear combination of z and $\overline{x}$, weighted by their respective relative precisions (γ_x and γ_z).

Second, at the beginning of period 2, new information comes into the market. This fact means that the updating of α_{i2} by the firm will come after the observation of z and q_1. On the other hand, consumers' update of beliefs comes after the observation of p_2 from an inference on z, and their previous experience α_{i1}.

The Bayesian updating in period 2 is,

$$E\{\alpha_{i2}|\alpha_{i1},z\} =$$
$$E\{\alpha_{i2}\}+\begin{pmatrix} Cov(\alpha_{i2},\alpha_{i1}) & Cov(\alpha_{i2},z)\end{pmatrix}\begin{pmatrix} Var(\alpha_{i1}) & Cov(\alpha_{i2},\alpha_{i1}) \\ Cov(\alpha_{i2},\alpha_{i1}) & Var(z)\end{pmatrix}^{-1}\begin{pmatrix}\alpha_{i1}-E\{\alpha 1\} \\ z-E\{z\}\end{pmatrix}. \tag{A.4}$$

Let us calculate the expected values, variances and the variance-covariance matrix:

1. Recall that expected values are $E\{\alpha_{i2}\} = E\{\alpha_{i1}\} = E\{z\} = \overline{x}$.
2. Variances are $Var(\alpha_{i1}) = \sigma_x^2+\sigma_\omega^2+\sigma_\upsilon^2$, $Var(\alpha_{i2}) = \sigma_x^2+\sigma_\omega^2$ and $Var(z) = \sigma_z^2$.
3. It is important to note that the updating of α_{i2} is conditional to α_{i1} and z i.e., we have three random variables distributed normally and correlated, where the variance-covariance matrix is as follows:
$$\begin{pmatrix}\alpha_{i1}\\ \alpha_{i2}\\ z\end{pmatrix}\sim N\left(\begin{pmatrix}\overline{x}\\ \overline{x}\\ \overline{x}\end{pmatrix},\begin{pmatrix}\sigma_\alpha^2 & \sigma_x^2+\sigma_\omega^2 & \sigma_x^2\\ \sigma_x^2+\sigma_\omega^2 & \sigma_\alpha^2 & \sigma_x^2\\ \sigma_x^2 & \sigma_x^2 & \sigma_z^2\end{pmatrix}\right). \tag{A.5}$$
4. Substituting in (A.4) all the terms above, we get the following expression:
$$E\{\alpha_{i2}|\alpha_{i1},z\} = \overline{x}\left(1-\frac{\sigma_x^2\sigma_\varphi^2+\sigma_\omega^2\sigma_z^2}{|\Sigma|}-\frac{\sigma_x^2\sigma_\upsilon^2}{|\Sigma|}\right)+\alpha_{i1}\left(\frac{\sigma_x^2\sigma_\varphi^2+\sigma_\omega^2\sigma_z^2}{|\Sigma|}\right)+z\left(\frac{\sigma_x^2\sigma_\upsilon^2}{|\Sigma|}\right),$$
where $|\Sigma| = \sigma_\alpha^2\sigma_z^2-\sigma_x^4$ is the determinant of the variance-covariance matrix above specified. Let
$$\delta_\alpha = \frac{\sigma_x^2\sigma_\varphi^2+\sigma_\omega^2\sigma_z^2}{\sigma_\alpha^2\sigma_z^2-\sigma_x^4},$$
$$\delta_z = \frac{\sigma_x^2\sigma_\upsilon^2}{\sigma_\alpha^2\sigma_z^2-\sigma_x^4},$$
and
$$\delta_x = 1-\delta_\alpha-\delta_z,$$
and substituting above, we get the consumers' Bayesian updating of their privacy concerns in period 2, conditional on z and α_{i1} .

$$E\left\{\alpha_{i2}|\alpha_{i1},z\right\}=\overline{x}\delta_x+\alpha_{i1}\delta_\alpha+z\delta_z.$$

In these equations $\sigma_\alpha^2=\sigma_x^2+\sigma_\omega^2+\sigma_\vartheta^2$ and $\sigma_z^2=\sigma_x^2+\sigma_\varphi^2$. Therefore, we can rewrite,

$$\delta_z=\gamma_z\left(1-\delta_\alpha\right).$$

$$\delta_x=\left(1-\gamma_z\right)\left(1-\delta_\alpha\right).$$

Appendix B: Proofs

Proof of Proposition 1

We prove here that the uniform equilibrium specified in the main text is the unique equilibrium where comsumers' inferences about z are a differentiable and invertible function of p_2^u.

To see that, note that the calculations show that this a linear equilibrium. The uniqueness property follows from the nature of the signaling differential equation. Assume that consumers infer $z=\hat{z}(p_2^u)$ if second period price is p_2^u, where $\hat{z}$ is C^1. The demand curve faced by the firm in period 2 is

$$\lambda\left(\theta_i-p_2^u\right)+\left(1-\lambda\right)\left(\theta_i-\left(\overline{x}\gamma_x+\delta_z\hat{z}p_2^u+z\left(p_2^u\right)\delta_\alpha\gamma_z\right)-p_2^u\right).$$

The profit maximization price satisfies the first-order condition

$$\begin{aligned}(1-\lambda)(r+1)\left(-z\left(p_2^u\right)\delta_\alpha\gamma_z+\theta_i-p_2^u-x\gamma_x-\delta_z\hat{z}'p_2^u-\hat{z}\left(p_2^u\right)\delta_z\right)&\\+\lambda(\theta_i-p_2^u)-\lambda p+(1-\lambda)(-p_2^u)(1+r)&=0,\end{aligned}$$

and implicitly defines the correct rule, $z(p_2^u)$. In a Bayes-Nash equilibrium, consumers use the correct inference rule, that is $\hat{z}\left(p_2^u\right)=z\left(p_2^u\right)$; hence, $z\left(p_2^u\right)$ must solve the ordinary differential equation

$$\begin{aligned}(2p_2^u-\theta_i)(1-r(1-\lambda))+\overline{x}\gamma_x(1+r)(1-\lambda)=&\\-\left(1+r\right)(1-\lambda)\left(z'p_2^u\delta_z+z\left(p_2^u\right)\left(\delta_z+\delta_\alpha\gamma_z\right)\right)&.\end{aligned}$$

We proceed by ordering and simplifying the terms in the previous differential equation to look for general/particular solutions. To that end,

- Firstly, dividing the ordinary differential equation by $(1+r)(1-\lambda)p_2^u\delta_z$, we get

$$\begin{aligned}\frac{2}{\delta_z}\left(\frac{1+r\left(1-\lambda\right)}{\left(1+r\right)\left(1-\lambda\right)}\right)-\frac{\theta_i}{p_2^u\delta_z}\left(\frac{1+r\left(1-\lambda\right)}{\left(1+r\right)\left(1-\lambda\right)}\right)+\frac{\overline{x}\gamma_x}{\delta_z}=&\\-\left(\frac{z'\left(p_2^u\right)\delta_zp_2+z\left(p_2^u\right)\left(\delta_z+\delta_\alpha\gamma_z\right)}{p_2^u\delta_z}\right)&.\end{aligned}$$

Letting $s = \frac{2}{\delta_z}\left(\frac{1+r(1-\lambda)}{(1+r)(1-\lambda)}\right)$, $m = \frac{\theta_i}{\delta_z}\left(\frac{1+r(1-\lambda)}{(1+r)(1-\lambda)}\right)$, $t = \frac{\overline{x}\gamma_x}{\delta_z}$,and $r = \frac{(\delta_z+\delta_\alpha\gamma_z)}{\delta_z}$, and reordering the terms yields

$$z^{'}\left(p_2^u\right) + z\left(p_2^u\right)p_2^{-1}r = p_2 m^{-1} - p_2^{-1}t - s.$$

- Secondly, multiplying the above expression by p^r (the integrating factor) then gives

$$p^r\left(z^{'}\left(p_2^u\right) + z\left(p_2\right)p_2^{-1}r\right) = p^r\left(mp_2^{-1} - tp_2^{-1} - s\right),$$

  which may be integrated to

$$p^r\left(z\left(p_2\right)\right) = p^r\left(\frac{m-t}{r} - \frac{p_2 \quad s}{1+r}\right) + C.$$

  for some constant C. This is a general solution. To determine C, we need the value of the function $z(p_2)$ at one point. For instance, if $z(0)$ is finite (the initial condition), then, evaluating the differential equation at $p_2 = 0$, gives that $C = 0$. Hence $z(p_2)$ is linear in p_2.

Proof of Proposition 2

The calculations show that this a linear equilibrium. The uniqueness property follows from the nature of the signaling differential equation. Assume that consumers infer $z = \hat{z}(p_{2C})$ if second period price is p_{2C}, where $\hat{z}$ is C^1.

The demand curve faced by the firm in period 2 is

$$(1-\lambda)p_{2C}(1+r)\left(\theta_i - z\delta_\alpha\gamma_z - \overline{x}\gamma_x - p_{2C} - \hat{z}\left(p_{2C}\right)\delta_z\right) + \lambda p_{2B}(\theta_i - p_{2B}).$$

The profit maximization price satisfies the first-order condition

$$(1-\lambda)(1+r)\left(\theta_i - p_{2C} - \overline{x}\gamma_x - z\delta_\alpha\gamma_z - \hat{z}\delta_z\right) - (1-\lambda)p_{2C}(1+r)\left(1 + \hat{z'}\left(p_{2C}\right)\delta_z\right) = 0,$$

and implicitly defines the correct rule, $z(p_{2C})$. In a Bayes-Nash equilibrium, consumers use the correct inference rule, that is $\hat{z}\left(p_{2C}\right) = z\left(p_{2C}\right)$; hence, $z\left(p_{2C}\right)$ must solve the ordinary differential equation

$$\begin{aligned}(\theta_i - \overline{x}\gamma_x)\left[(1-\lambda)(1+r)\right] - 2p_{2C}\left[(1-\lambda)(1+r)\right] = \\ \left[(1-\lambda)(1+r)\right]\left(z\left(\delta_z\gamma_z + \delta_z\right) + z'p_{2C}\delta_z\right).\end{aligned}$$

We proceed by ordering and simplifying the terms in the previous differential equation to look for general/particular solutions. To that end,

- Firstly, dividing the ordinary differential equation by $(1+r)(1-\lambda)p_{2C}\delta_z$, we get

$$\frac{(\theta_i - \overline{x}\gamma_x)}{p_{2C}\delta_z} - \frac{2}{\delta_z} = \frac{z\,(\delta_z\gamma_z + \delta_z)}{p_{2C}\delta_z} + z'.$$

  Letting $m = \frac{(\theta_i - \overline{x}\gamma_x)}{\delta_z}$, $s = \frac{2}{\delta_z}$, and $r = \frac{(\delta_z\gamma_z + \delta_z)}{\delta_z}$, and reordering terms yields

$$z' + zp_{2C}^{-1}r = mp_{2C}^{-1} - s.$$

- Secondly, multiplying the above expression by p^r (the integrating factor) then gives

$$p^r\left(z^{'}\left(p_{2C}\right) + z\left(p_{2C}\right)p_{2C}^{-1}r\right) = p^r\left(mp_{2C}^{-1} - s\right),$$

  which may be integrated to

$$p^r\left(\frac{m}{r} - \frac{p_{2C}s}{1+r}\right) + C,$$

  for some constant C. This is a general solution. To determine C, we need the value of the function $z(p_{2C})$ at one point. For instance, if $z(0)$ is finite (the initial condition), then, evaluating the differential equation at $p_{2C} = 0$, gives that $C = 0$. Hence $z(p_{2C})$ is linear in p_{2C}.

Proof of Proposition 3

We want to know whether channel-based price discrimination generates higher second period profits than those of uniform pricing. First, from Proposition 1, we have that under uniform pricing, first-period expected profits are given by

$$E\Pi_1^{u*} = \frac{(\theta_i + (1-\lambda)(\theta_i r - \overline{x}\,(1+r))^2}{4(1+(1-\lambda)r)}$$

and second-period expected profit by,

$$E\Pi_2^{u*} = \frac{(2-\delta_\alpha)\,\delta_\alpha\,(\theta_i(1+(1-\lambda)r) - (1-\lambda)(1+r)\,(\overline{x}\gamma_x + \hat{z}\gamma_z))^2}{4\,(1+(1-\lambda)\,r)}.$$

Similarly, from Proposition 2, first period expected profits under price discrimination are given by,

$$E\Pi_1^{d*} = \frac{1}{4}\left(\theta_i^2\,(1+r\,(1-\lambda)) + 2\theta_i\overline{x}\,(1-\lambda)\,(1+r) + \overline{x}^2\,(1-\lambda)\,(1+r)\right)$$

and second-period expected profits by,

$$E\Pi_2^{d*} = \frac{1}{4}\left((1-\lambda)(1+r)\,(2-\delta_\alpha)\,\delta_\alpha\,(\theta_i - (\overline{x}\gamma_x + \hat{z}\gamma_z))^2 + \theta)^2\lambda\right).$$

In order to facilitate the computations, let us define the following terms:

- $A = (1 - \lambda)(1 + r)$,
- $B = (1 + r(1 - \lambda))$,
- $C = \overline{x}\gamma_x + z\gamma_z$,
- $D = (2 - \delta_\alpha)\delta_\alpha$,
- $B - A = \lambda$,
- $1 - D = 1 - (2 - \delta_\alpha)\delta_\alpha$,

which are all non-negative. Then, we rewrite $E\Pi_2^{d*} - E\Pi_2^{u*} > 0$ using the above equations and the terms defined before as:

$$\frac{1}{4}\left(AD(\theta_i - C)^2 + \theta_i{}^2\lambda\right) > \frac{D(\theta_i B - AC)^2}{4B}.$$

Thus, operating and simplifying,

$$AD(\theta_i - C)^2 + \theta_i^2\lambda > \frac{D(\theta_i B - AC)^2}{B};$$

$$ADB(\theta_i - C)^2 + \theta_i^2\lambda B > D(\theta_i B - AC)^2;$$

$$ADB\left(\theta_i{}^2 + C^2 - 2\theta_i C\right) + \theta_i^2\lambda B > D\left(\theta_i{}^2 B^2 + A^2C^2 - 2\theta_i BAC;\right)$$

$$\theta_i{}^2\lambda B > D\left(\theta_i{}^2 B(B - A) - AC^2(B - A)\right);$$

$$\theta_i{}^2\lambda B > D\left(\theta_i{}^2 B\lambda - AC^2\lambda\right);$$

$$\theta_i{}^2\lambda B > D\lambda\left(\theta_i{}^2 B - AC^2\right);$$

$$\theta_i{}^2 B > D\left(\theta_i{}^2 B - AC^2\right);$$

$$\theta_i{}^2 B > \theta_i{}^2 BD - AC^2 D;$$

Finally, we get that the inequality is positive, that is,

$$\theta_i{}^2 B(1 - D) + AC^2 D > 0.$$

Then, the second period expected profits from channel-based price discrimination are higher than those from uniform pricing in the two channels.

Next, we show that channel-based price discrimination generates higher aggregate expected profits than those of uniform pricing. Let $E\Pi_a^{d*}$ and $E\Pi_a^{u*}$, be aggregate expected profits under price discrimination and uniform pricing, respectively. Then, using the above notation, rewrite $E\Pi_a^{d*} - E\Pi_a^{u*} > 0$ as,

$$\frac{1}{4}\left(AD(\theta_i - C)^2 + \overline{x}^2 A - 2\theta_i A + {\theta_i}^2 \lambda + \theta_i^2 B\right) > \frac{D(\theta_i B - AC)^2 + (\theta_i - \overline{x}\,(B-\lambda))^2}{4B}.$$

Thus, operating and simplifying,

$$\frac{1}{4}\left(\frac{A\lambda\left(DC^2 + \overline{x}^2\right)}{B} + \lambda(\delta_\alpha - 1)^2\theta_i^2\right) > 0.$$

Given that all the terms are non-negative, we get that the aggregate expected profits are higher under price discrimination than under uniform pricing.

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