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Tax Federalism and Cooperative Games: Stability Analysis

>Emilio Calvo
Universitat de València, ERI-CES, Spain

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TAX FEDERALISM AND COOPERATIVE GAMES: STABILITY ANALYSIS

Emilio Calvo[1]
Department of Economic Analysis and ERI-CES.
Universitat de Valencia, Spain.

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Abstract
This paper is devoted to analyzing the problem of how to distribute public spending among the different regions of a country once all taxes are collected. We model the problem as a cooperative game in coalitional form. For that purpose, we need to specify how much tax is collected in every region (and coalition of regions) in the country under secession. In this way, we obtain the tax game of the problem, and its core is given by the set of stable tax allocations. Following such an approach, we are able to analyze the stability of a tax financing system. The Spanish case is considered and we show that the present regional financial system is unstable from this perspective.

Keywords: fiscal federalism; fiscal stability; secessionism; coalitional games.

JEL Classification: H72, H77, C71

[1] E-mail: Emilio.Calvo@uv.es
ORCID ID: 0000-0002-7726-4325

1 INTRODUCTION

One of the main issues in fiscal federalism is how to distribute the total public budget between central government and regional authorities, once all tax has been collected in the country. As regions enter into the discussion as actors bargaining over the distribution of funds, the decision of how much should be spent in each region depends more and more on local government. Local institutions want to decide not only on what and how to spent the public budget assigned to their region, but also on *how much* to spend according to the citizens' needs.
With such a backdrop, in the search for a well-functioning regional financing system, three basic principles appear recurrently[2]:

- *Non-discrimination*: Distributed funds must provide a uniform level of public services throughout the country.

- *Fairness in redistribution*: Allocation of funds should vary *directly* depending on fiscal needs and *inversely* with the tax capacity of each jurisdiction.

- O*rdinality*: The results of the equalization should be tolerable for donors and recipients alike. They should narrow financing disparities across regions without altering their per capita wealth relative ranking. They should not carry equalization beyond a generally acceptable level.

However, some problems appear if we want to make all these principles fully compatible. Firstly, when part of the public budget is decentralized, regional administrations may disagree on how needs should be measured (health, education, social services,). They may have different priorities and, hence, different spending policies. Enforcing a uniform level of public services across regions forces regional administrations to become 'policy takers', in contradiction with their aspiration to determine their own expending policy. See for example the discussion in King and Eiser (2016) of the Barnett formula, which is applied in the UK for allocating grants to Scotland, Wales and Northen Ireland.
In addition, wide disparities between regions in their per capita rent could it make difficult to build a financial rule compatible with the non-discrimination principle. It could imply high transfers of rent, a source of possible disappointment for richer regions. In some countries, where richer regions have also strong secessionist aspirations, rent transfers are usually seen as exploitation by central political authorities. Secession ideologists often claim that their citizens would be better off economically if they lived in an independent country[3]. Given this political scenario, with increased tension of secessionist aspirations, exit treats can be a useful bargaining tool in negotiations over the distribution of the tax budget. Central government may be willing to send disproportionately large transfers to such regions to buy their 'cooperation'. However, such discriminatory behavior is a permanent source of instability of the system, feeding the same secessionist aspirations in discriminated regions.
Therefore, *stability* is a desirable property that should be taken into account when designing a regional financial system. That is, the tax system should minimize the grievances of a region, or a group of regions, against the total budget obtained with such a financing rule. Obviously, to measure such grievances implies finding an estimation of the difference between (i) how much a region would obtain with the rule and (ii) how much would be at their disposal under secession.
Up to now, *Fiscal Balances* (FB) had been used mostly for the analysis of such grievances. In a decentralized country, the fiscal balances determine the differences between the total public

[2] We quote these principles (among others) from De la Fuente, Thöne and Kastrop (2016).
[3] The secessionist ideology includes religious, cultural and political principles. However, we focus the discussion on economic terms only.

spending allocated to each region and the total amount of taxes collected. Increasing attention has been paid by academics to this topic. Computation of FB's can be found for Spain in Castells *et al.* (2000), Uriel and Barberán (2007), López-Casanovas and Rosselló-Villalonga (2014), and De la Fuente *et al.* (2014). For other countries, Italy is considered in Ferraro and Zanardi (2011), and in Giannola *et al.* (2016); the United Kingdom has been considered in McLean and McMillan (2003), and Oxford Economics (2008); Ireland in Morgenroth (2010); the US in Dubay (2006); and Canada in Ruggeri (2010).

Comparative studies of FBs across several countries are made in De la Fuente (2014), where Catalonia is compared with similar regions in other countries, and in Monastell and Sánchez (2012), where territorial redistribution of the public budget is analyzed.

Unfortunately, FBs cannot give us a clear idea of what could happen in a secessionist scenario. It is clear that under secession of a region, or a group of regions, the total taxes collected cannot be the same when regions remain united. Clearly, the commercial relationship between the secessionist area and the other regions that remain in the country would change in intensity. This is the well-known *border effect*. It establishes that domestic agents trade more with each other than with foreign agents of the same size and distance. The range of assumptions that can be made to determine the intensity in this effect can go from a totally friendly process to a more traumatic result. In addition, in a secessionist scenario, new borders appear with their corresponding custom tariffs. Therefore, on one hand, indirect taxes on inter-regional exports of the region are lost, because now they are converted into exports to a foreign country. On the other hand, indirect taxes on inter-regional imports are gained, because now they are converted into imports from abroad.

In order to analyze the stability of a tax financial system we follow a cooperative game theoretic approach. In this paper and its companion (Calvo, 2018), we propose an analytic tool to measure the tax budget which a region or group of regions by themselves can manage, without help of the remaining regions, in case of secession. Such an analytic tool is a coalitional *tax game* in transferable utility form (N, v_t), where N is the set of regions of the country, and $v_t(S)$ is the total tax budget at disposal of each possible coalition of regions $S \subseteq N$.

Insofar, as the tax game (N, v_t) is at our disposal, we can determine the set of *stable allocations*. That is, the set of tax budget distribution between regions that cannot be objected to by any coalition of regions, in the sense that they could obtain a greater tax amount than the allocation gives to them. This is the familiar concept of the core of a cooperative game. A tax financial rule will be stable if the allocated grants between regions belong to the core of the tax game.

Following this Introduction, in Section 2, we propose a way to build the tax game (N, v_t). For that aim, we use the tax matrix $T = [t_{ij}]$, where t_{ii} is the sum of all direct and indirect taxes collected from the products of region i sold within the region; plus the direct taxes from exports abroad; as well as the indirect taxes applied to the imports from abroad. And for each $j \in N \backslash i$, t_{ij} is the sum of all direct and indirect taxes from the production of goods and services of region i sold in region j. In the national accounts data usually are aggregate by regions; hence, for each region i, if we wish to obtain a disaggregated account vector $T_i = (t_{i1}, \ldots, t_{in})$ we need to make and *indirect* estimation. The *trade matrix* $C = [c_{ij}]$, which measures the flow of goods and services between regions of a country, will help us to estimate these tax coefficients. The pair (N, T) with set of regions N and tax matrix T will be called a *tax problem*.

In Section 3, given a tax problem (N, T), we build its associated *tax game* (N, v_t), by defining $v_t(S) = \sum_{i,j \in S} t_{ij}$, for each coalition $S \subseteq N$. The worth $v_t(S)$ is the sum of all collected taxes in each region of S, derived only from trade between regions of S and with foreign countries. These are the total collected taxes that the coalition of regions S can manage without the help of the remaining regions, $N\backslash S$, in the country. A tax allocation will be a way to share the total collected taxes between the regions of N. We define the set of stable tax allocations as the core of the tax game, denoted by $C(N, v_t)$. This is a *pessimistic* approach, because in the definition of v_t we assume that all commercial relationships between regions of S and $N\backslash S$ are broken. We relax this assumption later, but with this conservative pessimistic criterion, we guarantee that any tax allocation out of the core will be unstable for sure.

The main result of this Section 3 is of a positive nature: *the set of stable tax allocations* $C(N, v_t)$ *is always nonempty for any tax problem* (N, T). This fact has a clear consequence of a political nature: it is always possible to design a *stable* financing regional system. However, when there are wide per capita rent disparities between regions, we could be in trouble if we would like to apply the first principle of non-discrimination. For example, consider the *population egalitarian rule*, which gives to each region a share of the total budget in *proportion with its population*, in such a way that the expenses per capita are the same in every region. We show in a simple numerical example, with three regions, that the population egalitarian rule can be unstable.

In Section 4, we analyze the Spanish case, with the data of 2014. We show that the *present regional financing system is unstable*. In particular, the Autonomous Community of Balearics can manage a greater budgetary amount by itself than it obtains with the present fiscal system, even in the worst of all possible scenarios (i.e. breaking all commercial relationships with the remaining regions of Spain, and losing all its associated taxes)[4]. However, Balearics is an exception from the point of view of stability. Presently, we are not able to find any other coalition of regions which can complain about the total tax budget obtained with the present financial tax system. Spain is a case of wide disparities in terms of per capita rent across regions. As we can expect, we show that the population egalitarian rule applied in Spain is also unstable. For this tax rule there is a large set of region coalitions which can complain against this tax allocation.

In Section 5 we relax the pessimistic approach in the definition of v_t, allowing some degree of trade relationships between the secessionist and remaining regions. Let $S \subseteq N$ be a coalition of regions that constitutes a new country by withdrawing from N. We assume that within the new country S the intensity of the intra-trade relationships does not change, and the commerce with the remaining regions $N\backslash S$ is reduced in proportion α. This implies a reduction in the same ratio of the direct taxes associated with the exports from S to $N\backslash S$ and the indirect taxes (custom tariffs) associated with the imports from $N\backslash S$ to S. The parameter α reflects the optimistic/pessimistic perception of this breakdown process. That is, the *intensity* of the border effect. Now the characteristic function v_t^α changes its definition accordingly. Being $\alpha = 0$ the most pessimistic case (and then $v_t^0 = v_t$), and $\alpha = 1$ the most optimistic case. We show that the set of stable tax allocations is always non-empty for every value of α.
We also define the *secessionist tax rule* ζ^α which computes how much tax every would-be independent region would expect to manage, for every scenario α. We call ζ^1 as the optimistic secessionist tax rule. We realize that ζ^1 is stable for any scenario α.
Additionally, we show that as the optimistic perception regarding the economy increases in case of secession (α increases), the set of stable tax allocations reduces accordingly, where ζ^1 the unique stable payoff when $\alpha = 1$. We can also use the value of α as an alternative measure of how dissatisfied a region is with respect to a given tax rule ψ. For every region i, let α_i^* be the value for which region i is indifferent to being independent (obtaining $\zeta_i^{\alpha^*}$) or remaining in the country (obtaining ψ_i). High values of α_i^* imply high optimistic expectations, such that the secessionist scenario compensates the present status quo. Equivalently, high values of α_i^*

[4] The same happens for years 2011, 2012, 1nd 2013.

means low dissatisfaction with respect to ψ_i.
We consider again the Spanish case. There are two regions with high secessionist aspirations: The Basque Country (PV) and Catalonia (Cat). It is quite remarkable that the Basque Country (PV) is *in a better position under the present system (called* AE *in this work) than it would be under secession, even in the most optimistic case* (ζ^1): $AE_{PV} = €24{,}449.6m > \zeta^1_{PV} = €21{,}301.6m$ The case of Catalonia (Cat) is different. In the most optimistic case it would improve its budget to $€9{,}754.3m$. If we calculate the value of α^*_{Cat} such that Catalonia is indifferent to either remaining in Spain or being independent, we obtain $\alpha^*_{Cat} = 0.473$. It is clear that, from an economic point of view, the secessionist aspirations of Catalonia do not need to be supported by a very optimistic scenario. So it looks rational to claim a change in the present regional financial status quo given by AE. Just the opposite case is autonomous community of Balearics (Ba). Even in the most pessimistic case, $\alpha = 0$, it should always prefer ζ^0, in which obtaining its individually rational payoffs it is in a better situation than with the present system AE.
In Section 6 we offer a non-cooperative support of the characteristic function v^α_t. For each pair of complementary coalitions, S and $N\backslash S$, a bargaining game over the distribution of the worth $v_t(N)$ of the grand coalition is considered. The intensity of the border effect, α_S and $\alpha_{N\backslash S}$, are the decision variables of the coalitions in case negotiations are broken. That is, α_S and $\alpha_{N\backslash S}$ are the values which coalitions threaten to play if negotiations break down without agreement. In the Literature[5] three different assumptions are made about the determination of these threats: *minimax values*, *rational threats*, and non-cooperative *Nash' equilibrium*. We observe that the pessimistic value v^0_t is supported by the minimax and the rational threats approaches. On the other side, Nash' equilibrium yields the optimistic value v^1_t.
Section 7 ends with some final comments and remarks.

2 THE TAX MODEL

Let $N = \{1,2,\dots,n\}$ be the set of regions in a country. We wish to specify the impact that trade between regions has on tax collection. This will be summarized by the *tax matrix* $T = [t_{ij}]$, where we assume $t_{ij} \geq 0$ for all $i,j \in N$[6]. The total amount $T(N)$ is the sum of all taxes collected within the country from all regions.
For each region in a country, the total taxes collected are determined by the economic activity within the region, trade with other regions, and the imports and exports from abroad. Usually the data of the national accounts are aggregate at most by regions. Then we know for each region i, the aggregate $T_i(N)$, which represents all taxes collected. This amount is the sum of direct taxes $D_i(N)$ plus indirect taxes $I_i(N)$. Hence, if we wish to obtain a disaggregated account vector $T_i = (t_{i1},\dots,t_{in})$ we need to make an indirect estimation. To this end we use the trade matrix $C = [c_{ij}]$ which measures the flow of goods and services between the regions of a country. That is, for each pair of regions $i,j \in N$, c_{ij} will be the amount of goods and services of region i sold to region j. Additionally, we denote by m_i total foreign imports of region i, and by x_i total foreign exports.
Direct taxes are determined by the production of goods and services sold within the region, by sales to other regions, and by exports abroad. Therefore, the disaggregation of these taxes among regions is made by:

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[5] See Myerson (1992), Chapter 9.2.

[6] Vector and matrix notation: let $X = [x_{ij}]$ be a square matrix. For each $i \in N$, we denote the row i by $X_i = (x_{i1},\dots,x_{in})$, and the column i by $X^i = (x_{1i},\dots,x_{ni})$; the sum of its components by $X_i(N) = \sum_{j\in N} x_{ij}$, and $X^i(N) = \sum_{j\in N} x_{ji}$. The sum of all components of the matrix by $X(N) = \sum_{i\in N}\sum_{j\in N} x_{ij}$. And for all $S \subseteq N$, $X(S) = \sum_{i\in S}\sum_{j\in S} x_{ij}$.

$$\begin{cases} D_{ii} = \dfrac{c_{ii} + x_i}{\sum_{j \in N} c_{ij} + x_i} D(N)_i, & i \in N, \\ D_{ij} = \dfrac{c_{ij}}{\sum_{j \in N} c_{ij} + x_i} D(N)_i, & j \in N \backslash i. \end{cases} \tag{1}$$

Indirect taxes are collected from the production sold within the region, by sales to other regions, and by imports from abroad (custom tariffs). Therefore:

$$\begin{cases} I_{ii} = \dfrac{c_{ii} + m_i}{\sum_{j \in N} c_{ij} + m_i} I_i, & i \in N, \\ I_{ij} = \dfrac{c_{ij}}{\sum_{j \in N} c_{ij} + m_i} I_i, & j \in N \backslash i. \end{cases} \tag{2}$$

The total taxes are $t_{ij} = D_{ij} + I_{ij}$. Hence, t_{ii} is the sum of all *indirect* taxes (VAT and excise duties on alcoholic beverages, energy products and electricity, and manufactured tobacco) collected from the products of region i sold within the region, as well as tariff and import duties applied to imports from abroad; along with *direct* taxes (from salaries, rents from capital, and corporate taxes) due to the production of goods sold within the region and exports abroad. For each $j \in N \backslash i$, t_{ij} is the sum of all direct and indirect taxes originated from the production of goods and services in region i sold to region j.

Given a set of regions N and a tax matrix $T \in \mathbb{R}_+^N$, the pair (N, T) is called a *tax problem*. The space of all tax problems with finite regions set N is denoted by $\mathcal{T}^{\mathrm{N}}$, and by $\mathcal{T}$ the space of all tax problems. A *tax allocation rule* ψ is a vector function $\psi: \mathcal{T}^{\mathrm{N}} \to \mathbb{R}^{\mathrm{N}}$, which for each problem (N, T) specifies how to redistribute all taxes collected among the regions.

An obvious property that a tax allocation rule should satisfy is that it must share among regions the full tax budget at their disposition. This property is called Efficiency.

Definition 3 *We say that a tax rule ψ on $\mathcal{T}^N$ is efficient if $\sum_{i \in N} \psi_i(N, T) = T(N)$.*

We define now two simple examples of tax rules. The first one is the *population egalitarian* rule, denoted by Eg. This rule yields to each region i a share of the total budget in *proportion with its population*, in such a way that the expenses per capita are the same in every region. Let $w \in \mathbb{R}_{++}^N$ be a vector of population, where w_i is the population of region $i \in N$.

Definition 4 *For any tax problem (N, T), the population egalitarian rule Eg is defined by*

$$Eg_i(N, T) = \frac{w_i}{w(N)} T(N), \qquad \forall i \in N.$$

The population egalitarian rule corresponds to the case of full solidarity between regions. Its purpose is that every citizen in the country should not be discriminated against with regard to public spending. In particular, there should not be any disparity in the per capita public budget between regions. Such an approach has a long traditional support on the left of the political spectrum. But it also fits well in the ideology of centralist supporters.

In the opposite extreme, we can define the *no transfers rule*, denoted by ξ.

Definition 5 *For any tax problem (N, T), the no transfers rule ξ is defined by $\xi_i(N, T) = T_i(N)$, for all $i \in N$.*

Here each region has at its disposal all taxes it has collected itself. Now, there is no solidarity at all between regions. Notice that the *Fiscal Balances* are the differences between the total public spending made in each region and the total taxes collected in it.

Definition 6 *For any tax problem* (N, T), *the fiscal balances* FB *associated to any tax rule* ψ, *is defined by* $FB(\psi)_i(N, T) = \psi_i(N, T) - T_i(N)$, *for all* $i \in N$.

Rich regions usually have negative fiscal balances, especially when there are wide differences between the per capita rent of the regions. If in addition, it also coincides with strong secessionist aspirations, these transfers of rent are presented as a sort of economic exploitation by central government. The FBs are presented as a measure of such grievances. Secession ideologists sometimes claim that their citizens would be richer if their region was an independent country, managing by themselves the total budget of ξ_i[7].
As we see with more detail in Section 7, the tax rule ξ yields a very naive measure of what happens under secession. In this breakdown scenario, changes appear in the inter-regional trade intensities (known as the 'border effect'), new custom tariffs, and modification in the collection of total indirect taxes. For this reason, we build the secessionist tax rule ζ in order to give a more accurate idea of what a region could obtain under secession.

3 STABLE ALLOCATIONS

Our interest is to compare different tax rules by their properties, such as *equity*, *solidarity*, *stability*, and so on. Properties that are more or less appealing can help to decide which rule to choose from. To this end we borrow concepts from cooperative game theory. There, *partial cooperation* between players is allowed, and then we can take into consideration what would happen where a subgroup of players (regions) decide to cooperate between themselves alone. We think that this methodology is especially suitable for the setting of fiscal federalism and fiscal balances.
For example, consider a country where some regions have secessionist aspirations. Such ambitions can be partially justified by a perception that there is a relatively unfair fiscal balance between the tax generated in the region and the total budget spent within it; or indeed by claiming that the citizens of this region would be economically even *better off* after secession rather than remaining within the country. A way to brave such a challenge is to build a tax rule which minimizes as far as possible the dissatisfaction that such a rule can produce among regions. An option to measure it is to compute the difference between what the rule gives to the group and what the group could obtain by themselves if they were to withdraw from the country. Therefore, we can try to design a tax rule where no region (or group of regions) would be any better off remaining than it would be breaking away from the country.
Alternatively, suppose that the central authority wishes to distribute the total budget in a fair way among regions. What "fair" means in this context is not altogether obvious. While one option is to spend the budget per capita equally in each region, another option is to take into account the contribution of each region to the tax budget of the country. One way to measure such a contribution is to compute the difference between the total taxes collected both with and without the region in the country.
In either of both scenarios mentioned before, we somehow need to account for total taxes that any group of regions can collect by themselves, without competing with any of the remaining regions in the country. This is just the kind of partial cooperation setting which is considered in the cooperative game theoretic approach.

We start with some basic definitions. A *transferable utility game* (TU-game for short) is given

[7] For example, recently in Spain the slogan '*Spain is robbing us*' was widely used in Catalonian nationalist propaganda.

by a pair (N, v) where N is a finite set of players, $|N| = n$, and $v: 2^N \to \mathbb{R}$ is a *characteristic function*, which assigns to every coalition $S \subseteq N$ a real number $v(S)$, satisfying $v(\emptyset) = 0$. The set 2^N denote the set of all subsets of N, and for all $S \subseteq N$, $v(S)$ is called the *worth* of S. We denote by (N, v) the *transferable utility game* with player set N and *characteristic function* v. The space of all games with finite player set N is denoted by $\mathcal{G}^N$, and by $\mathcal{G}$ the space of all games. Given a game (N, v) and coalition S, we write (S, v) for the subgame obtained by restricting v to subsets of S only (i.e., to 2^S).
A payoff allocation is any vector $x = (x_1, \ldots, x_n) \in \mathbb{R}^N$, where each component x_i is the utility payoff of player i. Given an allocation x we use the notation $x(S) = \sum_{i \in S} x_i$ for every $S \subseteq N$.
A *solution* is a function ψ which assigns a real number $\psi_i(N, v)$ to every game (N, v) and every player $i \in N$.
From the data of any *tax problem* $(N, T) \in \mathcal{T}^N$ we can associate a *cooperative tax game* as follows:

Definition 7 *For each coalition $S \subseteq N$, define $v_t(S) = \sum_{i,j \in S} t_{ij}$.*

The worth $v_t(S)$ is the sum of all taxes collected in each region of S, derived from the trade only between regions of S and with foreign countries. This constitutes total taxes that the coalition of regions S can collect without the help of the remaining regions, $N \backslash S$, in the country. We will say that $(N, v_t) \in \mathcal{G}^N$ is the *tax game* associated to the *tax problem* $(N, T) \in \mathcal{T}^N$.
An obvious property that a value ψ should satisfy is that it must share between players the full worth at their disposal. This property is called *Efficiency*. The definition in the context of cooperative games $\mathcal{G}^N$ or tax problems $\mathcal{T}^N$ is the same.

Definition 8 *We say that a solution ψ on $\mathcal{G}^N$ is efficient if $\sum_{i \in N} \psi_i(N, v) = v(N)$.*

We consider first the set of *stable* allocations. We say that an allocation x is *feasible for a coalition* S if $x(S) \leq v(S)$. So the players in S can achieve their components of x by dividing among themselves the worth of $v(S)$ that they can get if they cooperate together. The *excess of S in x*, denoted by $e(S, x, v)$, is defined by $e(S, x, v) = v(S) - x(S)$. The excess can be seen as a measure of the *dissatisfaction* of coalition S with the allocation x.
We say that a coalition S can *improve* an allocation x if there exists some allocation y such that y is feasible for S, $y(S) \leq v(S)$, and all players i in S get $y_i > x_i$. It is clear that in such a case $e(S, x, v) > 0$. An allocation x is in the *core* of v, called $C(N, v)$, if x is efficient and no coalition can improve on x (Gillies, 1953). That is,

$$C(N, v) = \{x \in \mathbb{R}^N | \ x(N) = v(N) \quad \wedge \quad e(S, x, v) \leq 0,\ \forall S \subseteq N, S \neq \emptyset\}.$$

Thus, an allocation x is not in the core, if there is some coalition S such that the players in S could improve on x by cooperating together and dividing the worth of $v(S)$ among themselves[8].
Core notions had played important roles in Economic Theory, as the core of a Walrasian economy, or the set of stable matchings. In our tax federalism setting, the core of the tax game will be the set of stable tax allocations.

Definition 9 *Given a tax problem (N, T), we say that an allocation rule ψ is stable if*

[8] The core set has the structure of a polytope. Core elements can be computed by linear programming techniques. The core can be a large set; or it can be empty. The characterization of games with nonempty cores was given in Bondareva (1962) and Shapley (1967).

$\psi(N, v_t) \in C(N, v_t)$.

Remark 1 The set of stable tax allocations can also be defined if preferred only in terms of tax problems. Let $(N, T) \in \mathcal{T}^N$ be a tax problem and coalition $S \subseteq N$. Denote by T_S the submatrix generated from T considering only the elements t_{ij} such that $i, j \in S$. Therefore $(S, T_S) \in \mathcal{T}^S$. A coalition S can improve the tax allocation x if there exists some tax allocation y such that y is feasible for S, $y(S) \leq T_S(S)$, and all players i in S get $y_i > x_i$. The core of a tax problem $(N, T) \in \mathcal{T}^N$, denoted by $C(N, T)$ is the set of all tax allocations which are efficient and no coalition can improve on. However, as we will make use extensively well-known results for the core in the TU-game setting, it will be convenient to work with the associated tax game $C(N, v_t)$.

Our first question is to solve the *existence* of stable tax allocations. Suppose that in a particular country its associated tax problem has an empty core. In such a case, we can deduce that any tax budget redistribution will be unsatisfactory for some regions. A predictable consequence will be the existence of a permanent secessionist tension in this country, impossible to solve by redistributive budget spending policies alone. It could be solved by improving the fiscal balances for all regions simultaneously, but at the expense of falling into wasteful fiscal debt, with harmful financial consequences in the long run.

However, the main result of this Section is of a positive nature: For every tax game *it is always possible to find stable tax allocations*.

This result opens the door for political cooperation because, from an economic perspective, staying together is always better that being apart. The source of this result is related with the familiar notion of increasing returns to scale. In our context, this property says that larger coalitions of regions have a relatively larger tax budget to share among them. This is the key fact, which guarantees that the set of stable tax allocations is always non-empty. Recall that a function with increasing returns to scale is characterized by increasing marginal contributions, which implies that the function is convex. The translation of the notion of convexity of a real function into the setting of cooperative games is the notion of *convex games*. Indeed, what we see is that, for any tax problem (N, T), its associated tax game (N, v_t) is convex, and hence its core (N, v_t) is non empty.

We briefly recall some previous notions and results on convex games. We say that (N, v) is a *convex game* if for all $S, T \subseteq N$ it holds that $v(S) + v(T) \leq v(S \cup T) - v(S \cap T)$. This condition is just the translation of the economic notion of increasing returns to scale, i.e. larger coalitions have a relatively larger worth. For any $i \in N$ and $S \subseteq N \backslash \{i\}$, the *marginal contribution* of player i to coalition S is defined by $m_i(S, v) = v(S \cup i) - v(S)$. It is also well-known that the convexity of a game is equivalent to the condition that $m_i(S, v) \leq m_i(T, v)$ for all $i \in N$ and $S \subsetneq T \subseteq N \backslash \{i\}$. This is just the convexity characterization by the increasing marginal contributions property. Convexity is a sufficient condition, which guarantees the existence of core allocations.

Theorem 10 *(Shapley, 1971) Every convex game (N, v) has a non-empty core, i.e. $C(N, v) \neq \emptyset$.*

Now we are ready to prove the main result of this Section.

Theorem 11 *Every tax problem (N, T) always has stable tax allocations.*

Proof Let (N,T) be a tax problem and (N,v_t) its associated tax game. Let $i \in N$ and $S \subsetneq T \subseteq N\backslash\{i\}$. The marginal contribution of player i to coalition S is given by

$$m_i(S,v_t) \coloneqq v_t(S \cup i) - v_t(S) = t_{ii} + \sum_{j\in S}(t_{ij} + t_{ji}), \qquad \forall S \subseteq N\backslash\{i\}.$$

Thus it is satisfied that

$$m_i(T,v_t) - m_i(S,v_t) = \sum_{j\in T\backslash S}(t_{ij} + t_{ji}) \geq 0,$$

and then, the tax game (N,v_t) is convex. Therefore, by Theorem 10 it holds that $C(N,v_t) \neq \emptyset$. ■

A useful consequence to (N,v_t) being a convex game, is that there is a simple way to obtain its core allocations. This can be done computing the marginal contributions vectors following a random order approach. Let a game (N,v), and let $\pi: N \rightarrow N$ be a permutation of the player set N. Denote by $\Pi(N)$ the set of all permutations defined in N. We interpret π as an order defined between players in N, i.e. each player i enter in position $\pi(i)$ in the order π. Denote by $P_\pi(i)$ the set of *predecessors of* i *in order* π, that is

$$P_\pi(i) \coloneqq \{j \in N: \ \pi(j) < \pi(i)\},$$

and define the *marginal vector* $m^\pi(N,v) \in \mathbb{R}^N$ by $m_i^\pi(N,v) := m_i(P_\pi(i),v)$, for all $i \in N$. It is immediate that

$$\sum_{i\in N} m_i^\pi(N,v) = v(N).$$

Hence, $m^\pi(N,v)$ is an efficient way to share the worth of the grand coalition. For convex games we have the following characterization result:

Theorem 12 *(Shapley, 1971; Ichiishi 1981) The game* (N,v) *is convex if and only if*

$$C(N,v) = \left\{\sum_{\pi\in\Pi(N)} \lambda_\pi m^\pi(N,v): \ \lambda_\pi \geq 0 \ \text{ and } \sum_{\pi\in\Pi(N)} \lambda_\pi = 1\right\}.$$

.

That is, the core is the convex hull of the set of marginal contributions vectors. In a tax game (N,v_t) it is easy to check that the components of the marginal contributions vectors $m^\pi(N,v_t)$ are given by

$$m_i^\pi(N,v_t) = t_{ii} + \sum_{j\in P_\pi(i)}(t_{ij} + t_{ji}), \qquad \forall i \in N.$$

We can ask now whether the egalitarian Eg, and the no transfers ξ, tax rules are always stable.

Firstly, we show that ξ is stable.

Proposition 13 *The no transfers rule* ξ *is stable on* $\mathcal{T}^N$.

Proof Note that $\sum_{i\in N}\xi_i(N,T) = \sum_{i\in N}T_i(N) = T(N)$, and hence ξ is efficient. Now, let a coalition $S \subseteq N$. Then it holds that

$$e(S,\xi,v_t) = v_t(S) - \sum_{i\in S}\xi_i(N,T) = \sum_{i\in S}\sum_{j\in S}t_{ij} - \sum_{i\in S}T_i(N)$$

$$= \sum_{i\in S}\sum_{j\in S}t_{ij} - \sum_{i\in S}\sum_{j\in N}t_{ij} = -\sum_{i\in S}\sum_{j\in N\backslash S}t_{ij} \leq 0.$$

Hence, $\xi(N,T) \in C(N,v_t)$. ■

With respect to the egalitarian rule Eg, stability depends on the problem at hand. If the differences in wealth between regions in a country are very high, it can come to pass that the full solidarity principle could yield unstable outcomes.

In what follows, we use a simple numerical example to illustrate the concepts defined up to now.

Example. Let $N = \{1,2,3\}$ be a country with three regions, and with the following tax matrix

$$T = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 4 & 3 \\ 1 & 3 & 5 \end{bmatrix}$$

The coefficient $t_{12} = 0$ comes from the fact that region 1 does not export goods and services to region 2. Therefore, there are no direct taxes, nor indirect taxes associated with the trade from region 1 to region 2. The associated tax game (N, v_t) is given by

$$v_t(1) = 2, v_t(2) = 4, v_t(3) = 5, v_t(\{1,2\}) = 2 + 0 + 2 + 4 = 8,$$

$$v_t(\{1,3\}) = 2 + 1 + 1 + 5 = 9, v_t(\{2,3\}) = 4 + 3 + 3 + 5 = 15, v_t(N) = 21.$$

We can check that the payoffs of the no transfers rule ξ are

$$\xi_1(N,T) = T_1(N) = 3, \xi_2(N,T) = T_2(N) = 9, \xi_3(N,T) = T_3(N) = 9.$$

To see the inequalities that satisfy any stable allocation x, note that, for region 1 it must hold that $x_1 \geq v_t(1) = 2$, and that $x_2 + x_3 \geq v_t(\{2,3\}) = 15$, which jointly with $x(N) = v_t(\{1,2,3\}) = 21$, implies that $x_1 \leq x(N) - v_t(N) = 21 - 15 = 6$. Making the same reasoning for regions 2 and 3, we realize that the core of the game (N, v_t) is given by the set

$$C(N,v_t) = \{(x_1,x_2,x_3) \in \mathbb{R}^3 : 2 \leq x_1 \leq 6; 4 \leq x_2 \leq 12; 5 \leq x_3 \leq 13\}.$$

This set turns out to be the convex hull of the following vertices:

$$C(N,v_t) = C_H\{(6,4,11), (4,4,13), (2,6,13), (2,12,7), (4,12,5), (6,10,5)\}$$

which by Theorem 11 are the marginal contributions vectors given in the next table:

π	m_1^π	m_2^π	m_3^π
123	2	6	13
132	2	12	7
213	4	4	13
231	6	4	11
312	4	12	5
321	6	10	5

We can represent graphically all these points with the help of the equilateral triangle given in Figure 1. This triangle represents all efficient and non-negative division payoffs. For example, for region 3, the bottom side represents allocations where region 3 obtains zero. The vertex (0,0,21) is the opposite case, where region 3 obtains the total taxes to share, i.e. 21. Each intermediate horizontal line represents a constant payoff for region 3 (between 0 and 21). The same happens for regions 1 and 2, with parallel lines of constant payoffs, between the maximum payoff of 21 in the vertex and the minimum 0 payoff on its opposite side.

The core $C(N, v_t)$ of this game is the shadow area of the hexagon in Figure 1, and we can see that the no transfer rule $\xi(N,T) = (3,9,9)$ is placed inside the core.

Now assume that the population of the three regions is given by the vector $w = (12,4,5)$. The payoffs of the population egalitarian rule are

$$Eg_1(N,T) = \frac{12}{21} \cdot 21 = 12, Eg_1(N,T) = \frac{4}{21} \cdot 21 = 4, Eg_1(N,T) = \frac{5}{21} \cdot 21 = 5.$$

This point $(3,9,9)$ is not in the core, because

$$e(\{2,3\}, Eg, v_t) = v_t(\{2,3\}) - (Eg_2 + Eg_3) = 15 - (4+5) = 6 > 0.$$

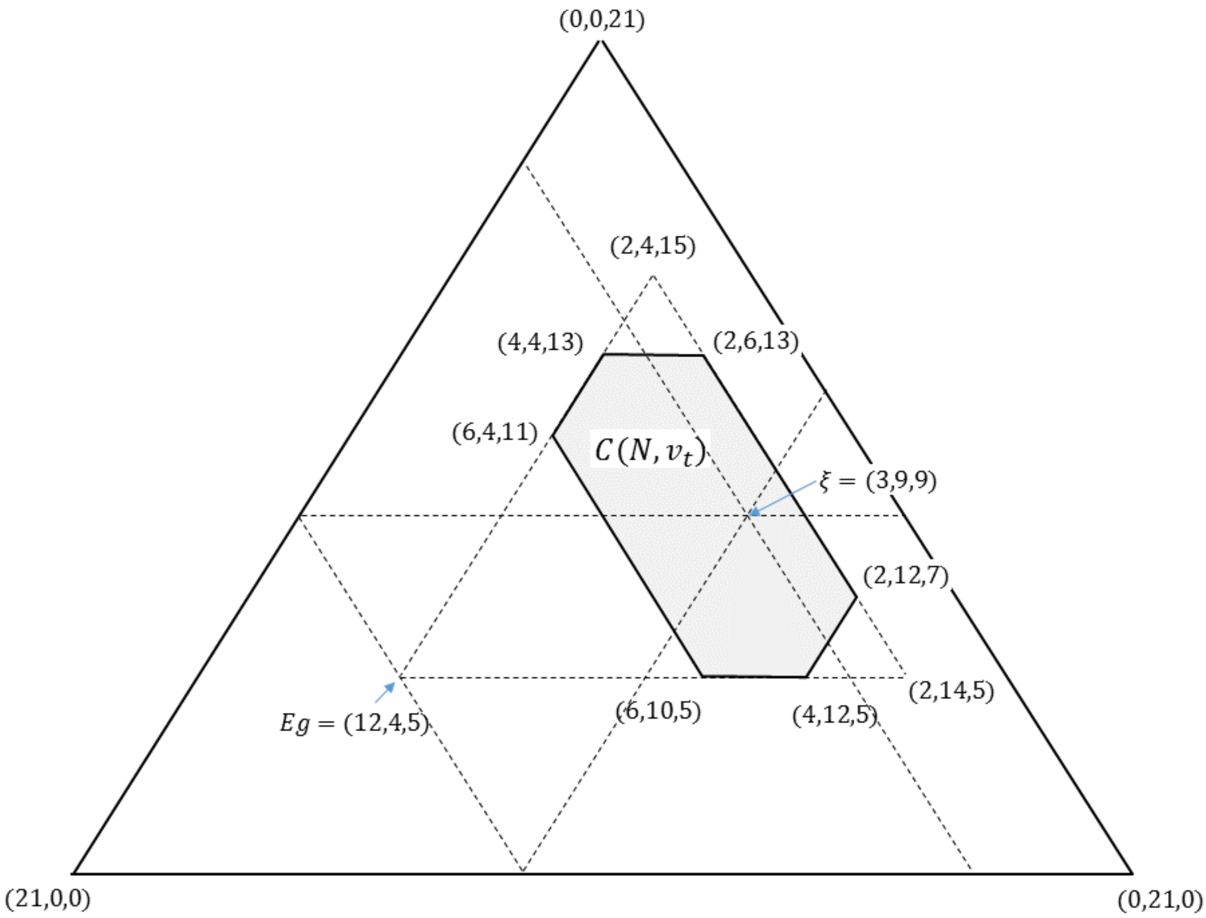

Figure 1 Efficient allocations

The stability of the population egalitarian rule will depend on the characteristics of the tax problem. It seems evident that big differences in the per capita tax revenues (T_i/w_i) increases the probability that Eg will yield unstable payoffs. On the contrary, similar ratios between regions increases the probability that Eg will be stable.

4 THE SPANISH CASE

We analyze the stability aspect of the tax system in the Spanish case with the help of its tax game. In this paper we use data from 2014, which we downloaded from the web page "Sistema de Cuentas Públicas Territorializadas" from "Ministerio Español de Hacienda y Administraciones Públicas"[9]. The results for 2011, 2012 and 2013 are very similar, and can be seen in the Excel file TaxFederalism.xlsx[10].

The fiscal flows caused by the tax activity between central and local governments in Spanish territories are taken as the starting point. From these data we obtain the net fiscal flows for each

[9] http://www.minhafp.gob.es/es-ES/CDI/Paginas/OtraInformacionEconomica/Sistema-cuentas-territorializadas.aspx

[10] https://www.uv.es/~ecalvo/TaxFederalism.xlsx

territory. To this end, public revenues are recorded on one hand (collected from taxes, fees, social security contributions, etc.), and public expenditures on the other (wages, investments, transfers, acquisition of current goods and services, public education and health, etc.). The net flow is the difference between the revenues and the expenditures in each territory, and this is known as *fiscal balance*. Several approaches are followed in the Literature in order to build these net flows. The most prominent are the "tax-benefit incidence" and the "monetary flow" approaches. For a good introduction on this topic see the IEB's Report on Fiscal Federalism and Public Finance 14 (2015). IEB Institut d'Economia de Barcelona. In lópez-Casanovas and Rosselló-Villalonga (2014) the FBs for Spain in 2005 are computed using both methodologies. The data used in our paper, an obtained from the "Sistema de Cuentas Públicas Territorializadas", are built following the tax-benefit approach. The reader can be found a more detailed explanation in De la Fuente *et al*. (2014).

Thousands euros		R	IT	DT	E	AE	FB	AFB	W
Autonomous Communities AC's	AC's	Total Tax Revenues	Indirect taxes	Direct taxes	Total Expenditures	Adjusted Expenditures	FB=E-R Fiscal Balance	AFB=AE-R Adjusted Fiscal Balance	Population
Andalucía	An	53.698.732	15.805.300	37.893.431	67.924.566	61.361.554	14.225.834	7.662.822	8.390.851
Aragón	Ara	11.343.657	2.775.122	8.568.536	13.215.938	12.176.964	1.872.281	833.307	1.328.334
Principado De Asturias	Ast	8.650.587	2.170.365	6.480.223	11.571.075	10.746.628	2.920.487	2.096.040	1.054.060
Illes Balears	Ba	9.600.424	2.434.579	7.165.845	8.943.515	8.067.125	-656.910	-1.533.299	1.120.470
Canarias	Cana	12.242.109	3.647.675	8.594.433	18.171.554	16.514.603	5.929.446	4.272.494	2.118.423
Cantabria	Cnt	4.824.547	1.221.096	3.603.450	5.796.789	5.338.254	972.242	513.707	586.240
Castilla y León	C-L	19.142.497	4.919.164	14.223.332	25.354.918	23.410.981	6.212.421	4.268.485	2.485.335
Castilla-La Mancha	C-M	13.785.329	3.840.842	9.944.486	17.028.809	15.411.625	3.243.480	1.626.296	2.067.580
Cataluña	Cat	70.376.301	16.940.305	53.435.996	66.330.218	60.542.524	-4.046.083	-9.833.776	7.399.601
Comunidad Valenciana	Va	36.155.746	9.913.803	26.241.943	38.305.965	34.436.335	2.150.219	-1.719.411	4.947.346
Extremadura	Ex	6.667.215	1.893.610	4.773.604	10.346.980	9.491.444	3.679.765	2.824.230	1.093.807
Galicia	Ga	20.250.535	5.482.765	14.767.770	26.074.726	23.932.122	5.824.191	3.681.587	2.739.332
Comunidad de Madrid	Ma	68.126.895	15.773.362	52.353.533	53.937.586	48.949.930	-14.189.309	-19.176.965	6.376.749
Región de Murcia	Mu	9.775.540	2.774.699	7.000.841	11.024.682	9.880.471	1.249.143	104.932	1.462.881
Comunidad Foral De Navarra	Na	5.731.657	1.428.491	4.303.166	6.344.956	5.847.499	613.300	115.842	636.003
País Vasco	PV	21.052.492	4.985.808	16.066.683	26.143.305	24.449.661	5.090.814	3.397.170	2.165.334
La Rioja	Ri	2.621.104	648.344	1.972.760	2.910.363	2.664.702	289.259	43.599	314.079
Ceuta y Melilla	CyMel	929.936	303.405	626.531	1.884.829	1.752.879	954.893	822.943	168.699
Total		*374.975.300*	*96.958.736*	*278.016.564*	*411.310.773*	*374.975.300*	*36.335.473*	*0*	*46.455.123*

Table 1 Tax revenues and expenditures 2014 (Thousands of €)

The data in the table shows that total expenditure is greater than total tax revenues, leading to a fiscal debt of 36.335.473. As we want to compare different tax rules, we will compute all of them assuming that the total net flow is in balance. For that reason, we adjust the expenditures in such a way that the total net balance will be zero. In this way, real flows are replaced by the so-called "neutralized" flows. For such an operation, we assume that this deficit is redistributed among the autonomous communities in proportion to their population[11], that is

$$AE_i = E_i - FB(N) \cdot \frac{w_i}{w(N)}, \qquad \forall i \in N.$$

The column of *Adjusted Expenditures* shows how the total expenses are spent in each AC. As can be seen in the *Adjusted Fiscal Balance* column, several communities have negative fiscal balance, so they are positive contributors to the wealth of the remaining regions. In the following we refer to this *Adjusted Expenditure* column as the present Spanish government tax rule AE. Its determination is closer to the product of continuous political bargaining than a

[11] The data of population considered in the "Sistema de Cuentas Públicas Territorializadas" do not coincide with the real data. They are adjusted in order to compensate for some geographical and demographic differences between regions. However, this modification of the population data forms part of the set of rules, which determine regional expenditures. In order to make a transparent comparison between alternative rules we prefer to use real population data.

clear and transparent formula. The reader can find a summary description in De la Fuente *et al.* (2016). A complete description is given in De la Fuente *et al.* (2014).
In order to see the stability of this rule, or any other we can consider (as the population egalitarian rule), we make an estimation of the tax matrix T for Spain.
For that purpose, we use the matrix of the commerce inter-regional trades provided by the c-interg[12] project institution, and the data for foreign imports and exports provided by the Spain Ministry of Industry, Commerce and Competitiveness, given on the Datacomex[13] website.

Interregional Trades	An	Ara	Ast	Ba	Cana	Cnt	C-L	C-M	Cat	Va	Ex	Ga	Ma	Mu	Na	PV	Ri	CyMel	Foreing Exports
An	24.561	228	390	899	4.783	72	2.190	2.055	5.216	3.069	2.680	565	2.879	2.923	207	623	82	1.072	*26.650*
Ara	286	7.588	153	31	172	189	1.073	854	3.956	1.730	57	271	1.195	81	582	1.350	141	5	*9.382*
Ast	287	255	5.408	7	39	689	1.976	61	174	185	146	682	163	76	296	369	342	1	*3.838*
Ba	59	0	54	3.632	59	0	5	32	232	71	0	69	42	7	0	3	0	5	*924*
Cana	2.273	25	7	7	5.846	6	77	23	1.516	73	2	26	105	9	8	27	1	2	*2.393*
Cnt	95	266	254	0	9	2.237	717	63	210	123	4	142	179	1	103	1.027	64	0	*2.546*
C-L	692	908	1.962	13	385	1.171	14.009	1.714	1.499	1.469	523	1.567	3.078	281	433	2.531	367	36	*12.752*
C-M	3.099	313	135	34	125	104	1.215	7.030	1.361	2.375	1.001	369	5.238	523	122	269	31	9	*5.399*
Cat	3.363	8.582	552	1.995	1.759	368	1.836	3.777	44.479	6.675	175	649	3.391	1.425	1.309	1.924	667	113	*60.291*
Va	1.966	1.133	80	1.242	331	278	820	2.184	4.028	21.907	119	250	1.988	1.753	159	578	134	24	*25.001*
Ex	1.323	12	38	0	20	13	420	341	155	131	3.450	74	1.005	59	13	41	13	1	*1.674*
Ga	1.495	555	2.522	171	642	738	3.690	353	1.252	656	35	17.306	1.079	92	110	1.036	6	14	*17.810*
Ma	3.072	1.063	684	317	1.367	383	2.260	3.085	3.409	2.587	507	1.615	11.600	820	442	1.489	167	114	*27.731*
Mu	2.533	64	59	85	188	28	133	679	605	3.644	96	232	619	4.921	59	90	15	20	*10.441*
Na	158	1.287	55	8	42	267	664	185	1.110	284	43	178	406	160	3.810	1.329	487	3	*8.141*
PV	419	1.421	343	58	72	730	3.505	327	1.839	749	179	1.133	633	64	1.822	15.809	414	6	*22.501*
Ri	214	157	12	0	13	28	1.143	95	267	656	14	59	278	5	472	812	2.172	1	*1.644*
CyMel	3	0	0	0	115	0	0	0	0	2	0	0	1	0	0	0	0	1	*38*
Foreing Imports	*30.958*	*8.553*	*3.342*	*1.455*	*3.556*	*1.861*	*12.244*	*5.974*	*72.221*	*21.373*	*980*	*14.413*	*50.864*	*11.960*	*4.163*	*17.131*	*1.154*	*642*	

Table 2 Interregional Trades+Imports+Exports 2014 (Millions of €)

Applying the definitions of t_{ij} given in Section 2, we obtain the tax matrix T for Spain:

Matrix [T]	An	Ara	Ast	Ba	Cana	Cnt	C-L	C-M	Cat	Va	Ex	Ga	Ma	Mu	Na	PV	Ri	CyMel
An	34.183.812	148.855	254.536	586.038	3.118.056	46.789	1.428.004	1.339.797	3.400.754	2.000.775	1.747.534	368.064	1.876.914	1.905.960	134.776	406.026	53.432	698.609
Ara	112.128	6.582.789	59.994	12.156	67.394	74.145	421.368	335.296	1.553.337	679.202	22.527	106.377	469.356	31.646	228.688	530.058	55.217	1.979
Ast	167.082	148.638	5.305.134	4.217	22.560	400.687	1.149.781	35.617	101.412	107.659	84.896	396.943	94.713	44.038	172.327	214.839	199.210	834
Ba	105.844	603	97.843	8.449.637	105.730	875	9.264	58.266	418.624	127.268	0	123.916	75.201	12.433	0	5.452	0	9.468
Cana	2.182.430	24.360	6.738	6.919	8.221.492	5.377	73.853	22.395	1.455.607	70.555	1.614	25.016	100.459	8.464	7.881	25.747	954	2.247
Cnt	58.391	163.448	156.124	0	5.370	2.823.982	440.218	38.755	128.977	75.296	2.619	87.357	109.806	674	63.257	631.048	39.054	169
C-L	292.824	384.227	829.780	5.504	162.823	495.180	11.262.982	724.726	633.816	621.124	221.407	662.953	1.301.892	118.849	183.347	1.070.457	155.226	15.381
C-M	1.477.622	149.175	64.344	16.254	59.619	49.541	579.350	6.002.255	648.917	1.132.368	477.236	175.916	2.497.926	249.226	58.289	128.054	15.008	4.229
Cat	1.620.548	4.135.756	265.802	961.643	847.482	177.213	884.729	1.820.254	51.793.956	3.216.814	84.574	312.777	1.634.155	686.737	630.762	927.345	321.339	54.415
Va	1.129.210	650.836	46.208	713.604	189.943	159.769	471.060	1.254.930	2.313.743	26.351.438	68.485	143.687	1.142.211	1.007.165	91.144	331.770	76.804	13.739
Ex	1.028.819	9.702	29.272	0	15.453	10.022	326.451	264.984	120.496	101.976	3.821.497	57.534	781.748	46.110	10.201	31.821	10.297	832
Ga	623.122	231.344	1.050.813	71.367	267.562	307.526	1.537.929	147.126	521.751	273.183	14.474	14.230.385	449.707	38.140	45.885	431.934	2.356	5.930
Ma	3.128.789	1.082.919	697.047	323.352	1.392.134	390.525	2.302.063	3.141.988	3.472.400	2.634.856	516.224	1.644.636	44.311.644	834.759	450.706	1.516.625	169.713	116.515
Mu	993.475	25.020	23.071	33.256	73.907	11.065	52.215	266.476	237.316	1.428.936	37.667	91.127	242.853	6.186.565	23.320	35.219	6.035	8.016
Na	51.818	423.037	18.007	2.505	13.709	87.649	218.305	60.790	365.046	93.350	14.094	58.660	133.393	52.517	3.540.699	436.781	160.153	1.145
PV	174.323	590.757	142.544	24.230	30.060	303.547	1.456.961	135.800	764.550	311.172	74.376	470.906	263.173	26.627	757.251	15.351.487	172.052	2.674
Ri	70.945	51.960	4.116	0	4.399	9.208	378.422	31.613	88.429	217.142	4.583	19.463	92.020	1.672	156.249	268.968	1.221.606	308
CyMel	13.083	1	0	0	496.722	2	965	10	785	7.113	1	88	2.176	53	53	1	0	408.881

Table 3 Tax Matrix 2014 (Thousands of €)

From the data of matrix T we can build the tax game (N, v_t).
We start analyzing the redistribution and stability properties of the Adjusted Expenditures (AE) rule.
Firstly, we realize that this rule is *unstable*, $AE \notin C(N, v_t)$. To see this fact, note that for the payoffs obtained by the AC of Balearics this rule is not even *individually rational*[14]. In a tax game, we have that $v_t(i) = t_{ii}$, and for Balearics it holds that

[12] http://212.227.102.53/explotacion_multidimensional_comercio_interregional/estadisticas.aspx
[13] http://datacomex.comercio.es/principal_comex_es.aspx
[14] Given a TU-game (N, v), a rule ψ is individually rational if $\psi_i(N, v) \geq v(i)$ for all $i \in N$.

$$v_t(Ba) = 8.449.637 > AE(Ba) = 8.067.125$$

This is a rather extreme situation, because it means that even in the worst of the possible scenarios (i.e. breaking all commercial relationships with the remaining ACs and losing all its associated taxes), this community can still manage a greater budgetary amount than it can obtain under the present fiscal system. The same happens for 2011, 2012, and 2013.

Nevertheless, Balearics is an exception from the point of view of stability. Actually, we are not able to find any other autonomous community, nor a coalition of ACs with a positive excess[15]. In particular, Catalonia, which has high secessionist aspirations, has a negative excess of

$$e(AE, \{Cat\}, v_t) = -8.748.568$$

And if we consider the set of 'Catalan Countries', formed by Catalonia (Cat), Valencia (Va), and Balearics (Ba), its excess is also negative

$$e(AE, \{Cat, Va, Ba\}, v_t) = -8.699.256$$

The redistributive effect of the AE rule can be seen using the second half of the following Table 4.

	AE	AFB	IR		GPD	IGDP	AEGDP		
AC's	*Adjusted Expenditures*	*Adjusted Fiscal Balance*	*Individually Rational*	$e(AE,\{i\})$ *IR-AE*	*Gross Domestic Product*	*Initial GDP GDP+R-AE*	*Final GDP IGDP+R-AE*	*IGDPpc*	*AEGDPpc*
An	61.361.554	7.662.822	34.183.812	-27.177.741	141.861.707	127.635.873	135.298.695	15,211	16,125
Ara	12.176.964	833.307	6.582.789	-5.594.175	33.186.731	31.314.451	32.147.757	23,574	24,202
Ast	10.746.628	2.096.040	5.305.134	-5.441.494	21.460.498	18.540.011	20.636.051	17,589	19,578
Ba	8.067.125	-1.533.299	8.449.637	382.512	26.865.799	27.522.709	25.989.410	24,564	23,195
Cana	16.514.603	4.272.494	8.221.492	-8.293.111	41.562.860	35.633.414	39.905.908	16,821	18,838
Cnt	5.338.254	513.707	2.823.982	-2.514.271	12.240.460	11.268.218	11.781.926	19,221	20,097
C-L	23.410.981	4.268.485	11.262.982	-12.147.999	54.035.706	47.823.285	52.091.770	19,242	20,960
C-M	15.411.625	1.626.296	6.002.255	-9.409.370	37.882.920	34.639.440	36.265.736	16,754	17,540
Cat	60.542.524	-9.833.776	51.793.956	-8.748.568	199.925.024	203.971.107	194.137.331	27,565	26,236
Va	34.436.335	-1.719.411	26.351.438	-8.084.897	99.438.227	97.288.007	95.568.596	19,665	19,317
Ex	9.491.444	2.824.230	3.821.497	-5.669.948	17.247.267	13.567.502	16.391.732	12,404	14,986
Ga	23.932.122	3.681.587	14.230.385	-9.701.737	54.710.029	48.885.838	52.567.425	17,846	19,190
Ma	48.949.930	-19.176.965	44.311.644	-4.638.286	197.819.018	212.008.327	192.831.362	33,247	30,240
Mu	9.880.471	104.932	6.186.565	-3.693.906	27.149.966	25.900.823	26.005.755	17,705	17,777
Na	5.847.499	115.842	3.540.699	-2.306.799	17.899.369	17.286.069	17.401.911	27,179	27,361
PV	24.449.661	3.397.170	15.351.487	-9.098.174	64.336.077	59.245.263	62.642.433	27,361	28,930
Ri	2.664.702	43.599	1.221.606	-1.443.096	7.856.850	7.567.591	7.611.189	24,095	24,233
CyMel	1.752.879	822.943	408.881	-1.343.998	2.990.491	2.035.598	2.858.541	12,066	16,945
Total	*374.975.300*	*0*	*250.050.242*		*1.058.469.000*	*1.022.133.527*	*1.022.133.527*	*22,003*	

Table 4 Adjusted Expenditures 2014

We look at the gross domestic product per capita as the wealth reference. Firstly, we need to compute what the GDP should be without the fiscal transfers made this year. This is what we call the Initial GDP[16]:

$$IGDP_i = GDP_i - FB_i, \qquad \forall i \in N$$

The Final GDP associated to any rule ψ is denoted by ψGDP and is equal to the Initial GDP plus the fiscal balance associated with it, that is the payoffs given by the rule ψ to the region minus the total tax revenues of the region, hence:

$$\psi GDP_i = IGDP_i + (\psi_i - R_i), \qquad \forall i \in N$$

[15] Note that the total number of possible coalitions is $2^{18} - 1$, and we have not checked all of them.

[16] Equivalently, this is the neutralized GDP in which we delete the fiscal debt for the year.

Finally, we can compare the Initial GDP per capita with final GDP per capita associated to ψ.

In the second hand part of Table 4 we can see this data for the present Spanish tax rule AE. The rule has some clear redistributive effects. In general, wealth per capita increases in regions where $IGDPpc$ is lower than average; and decreases in regions above average. However, we find some disturbing exceptions.
On one hand, we have the case of the Comunidad Valenciana (Va), whose $IGDPpc$ is lower than average, equal to €22.003; nevertheless, after redistribution, its final per capita rent decreases:

$$AvIGDPpc = 22.003 > IGDPpc_{Va} = 19.665 > AEGDPpc_{Va} = 19.317$$

On the other hand, regions that are richer than the average end up even richer than initially: La Rioja (Ri), Aragón (Ara), Comunidad Foral de Navarra (Na), and, above all, País Vasco (PV). This fact is illustrated in Figure 2, in which we can see the relation between the Adjusted Fiscal Balance per capita and the *IGDPpc*, where the negative slope of the regression line means an overall redistributive effect.

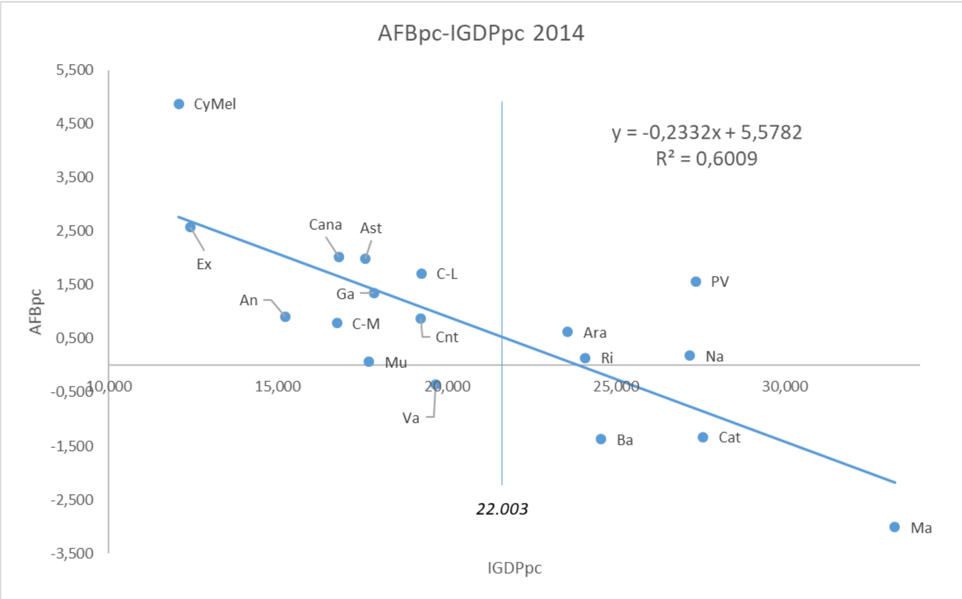

Figure 2

Besides, we realize that initial and final wealth rank[17] between ACs changes from [12.066 – 33.247] to [14.986 – 30.340]. This fact is shown in Figure 3.

The overall redistributive effect appears in the slope of the regression line, as it is less than 1 (i.e. 0.7668), but there are 13 pairs of ACs which interchange their wealth position ranking:

$$\left\{\begin{matrix}(CyMel, Ex), (CyMel, An), (Cana, Mu), (Ast, Mu), (Ast, Ga), (Ast, Va), (Ga, Va), \\ (Cnt, Va), (CL, Va), (Ara, Ba), (Ri, Ba), (Na, Cat), (PV, Cat)\end{matrix}\right\}$$

This clearly breaks the *ordinality* principle: The redistributive effect of a rule "should narrow financing disparities across regions without altering their wealth relative ranking."

[17] The difference between the maximum and minimum value.

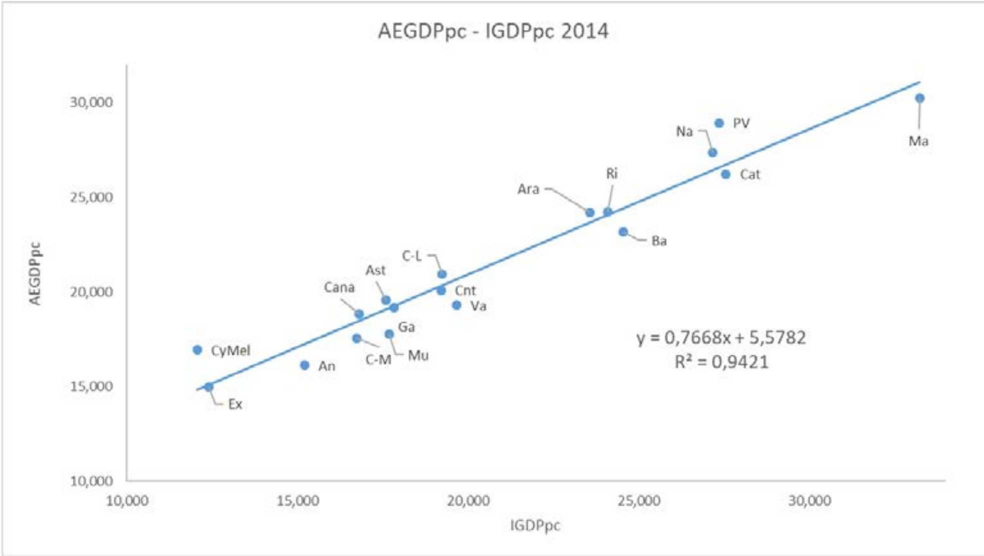

Figure 3

An alternative rule which has wide support, especially on the left of the political spectrum, is the *population egalitarian* rule Eg. Its purpose is that every citizen in a country should not be discriminated against in public spending. In particular, there should be no disparities in public spending per capita between the regions of a country.
This rule is defined for each region $i \in N$ by

$$Eg_i = \frac{w_i}{w(N)} \cdot AE(N)$$

We show in Table 5 the main data associated with the population egalitarian rule.

AC's	R Total Tax Revenues	Eg Population Egalitarian	EgFB=Eg-R Egalitarian Fiscal Balance	IR Individually Rational	$e(Eg,\{i\})$ IR-Eg	IGDP Initial GDP GDP+AFB	EgGDP Final EgGDP IGDP+EgFB	IGDPpc	EgGDPpc
An	53.698.732	67.729.065	14.030.333	34.183.812	-33.545.253	127.635.873	141.666.206	15,211	16,883
Ara	11.343.657	10.722.016	-621.641	6.582.789	-4.139.227	31.314.451	30.692.809	23,574	23,106
Ast	8.650.587	8.508.137	-142.451	5.305.134	-3.203.003	18.540.011	18.397.560	17,589	17,454
Ba	9.600.424	9.044.178	-556.246	8.449.637	-594.541	27.522.709	26.966.463	24,564	24,067
Cana	12.242.109	17.099.433	4.857.324	8.221.492	-8.877.941	35.633.414	40.490.738	16,821	19,114
Cnt	4.824.547	4.731.993	-92.553	2.823.982	-1.908.011	11.268.218	11.175.665	19,221	19,063
C-L	19.142.497	20.061.065	918.568	11.262.982	-8.798.082	47.823.285	48.741.853	19,242	19,612
C-M	13.785.329	16.689.038	2.903.709	6.002.255	-10.686.783	34.639.440	37.543.149	16,754	18,158
Cat	70.376.301	59.727.914	-10.648.387	51.793.956	-7.933.958	203.971.107	193.322.720	27,565	26,126
Va	36.155.746	39.933.865	3.778.119	26.351.438	-13.582.427	97.288.007	101.066.126	19,665	20,428
Ex	6.667.215	8.828.966	2.161.752	3.821.497	-5.007.470	13.567.502	15.729.254	12,404	14,380
Ga	20.250.535	22.111.274	1.860.739	14.230.385	-7.880.889	48.885.838	50.746.577	17,846	18,525
Ma	68.126.895	51.471.680	-16.655.216	44.311.644	-7.160.036	212.008.327	195.353.112	33,247	30,635
Mu	9.775.540	11.808.044	2.032.504	6.186.565	-5.621.479	25.900.823	27.933.328	17,705	19,095
Na	5.731.657	5.133.673	-597.983	3.540.699	-1.592.974	17.286.069	16.688.086	27,179	26,239
PV	21.052.492	17.478.088	-3.574.403	15.351.487	-2.126.601	59.245.263	55.670.860	27,361	25,710
Ri	2.621.104	2.535.172	-85.932	1.221.606	-1.313.566	7.567.591	7.481.659	24,095	23,821
CyMel	929.936	1.361.698	431.763	408.881	-952.817	2.035.598	2.467.360	12,066	14,626
Total	*374.975.300*	*374.975.300*	*0*	*250.050.242*		*1.022.133.527*	*1.022.133.527*	*22,003*	

Table 5 Population Egalitarian rule 2014

Note that there is a relatively wide disparity between the initial per capita rent of the Autonomous Communities in Spain. It goes from 12.066 to 33.247. The AC of Madrid (Ma) is 2.76 times richer than the AC of Ceuta and Melilla (CyMel). This big difference in wealth per capita allows us to guess that strong redistributive rules will induce unstable payoffs. As we might expect, we find that the Eg rule is unstable.
At first sight, it appears to be the contrary. Thus, we can check that Eg satisfies individual rationality (the column $IR - Eg$, in Table 5, has only negative values) contrary to the AE rule,

which yields a positive value for Balearics. However, we will find positive excesses when we consider coalitions with more than one element. In particular, consider the following coalition of ACs, which are in the bottom part of the $IGDPpc$ range, $S^* = \{CyMel, Ex, An, CM, Cana, Ast, Mu, Ga\}$. Now if we compute the excess of the remaining regions without S^*, that is coalition $N\backslash S^*$, its excess is positive:

$$e(N\backslash S^*, Eg, v_t) = v_t(N\backslash S^*) - Eg(N\backslash S^*) = 1.023.434$$

Hence, if we delete the set S^* of ACs from the tax system (losing $N\backslash S^*$ all commercial relationships with S^*) $N\backslash S^*$ will end up in a better position. This means that the transfer of fiscal rents which the Eg rule implies from $N\backslash S^*$ to S^* is enough to make the fiscal system unstable.

The interested reader can complete the exercise of computing this excess for coalitions $S' \subset S^*$ and will realize that in all these cases $e(N\backslash S', Eg, v_t) = v_t(N\backslash S') - Eg(N\backslash S') > 0$. These values can be easily computed in the associated Excel file TaxFederalism-X-M.xlsx.

In the following Figure 4 we see the relationship between egalitarian fiscal balance, $(Eg - R)pc$, and the $IGDPpc$.

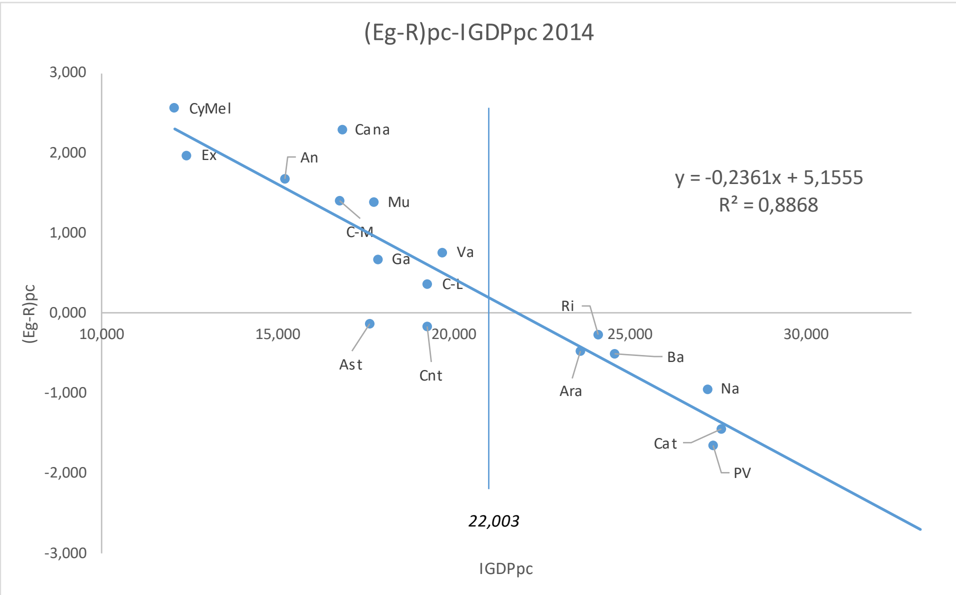

Figure 4

Note that the overall transfer effect in Eg is similar to the rule AR, as the slope for the Eg is negative -0,2361 and for AE is -0,2332. This can be seen also in Figure 5 that shows the relationship between the initial and final wealth per capita with the Eg rule.

Accordingly, the slope of the regression line for Eg is 0,7639 which is similar to the slope 0,7668 of the regression line for AE. The are 10 pairs of ACs which change their wealth ranking with the Eg rule:

$$\left\{\begin{matrix}(CyMel, Ex), (CM, Ast), (Mu, Ga), (Cana, Ast), (Cana, Mu), \\ (Cana, Ga), (Mu, Ga), (Mu, Cnt)(Na, PV), (Na, Cat)\end{matrix}\right\}$$

These are only slightly less than the 13 pairs generated by the AE rule. Hence we can say that the population egalitarian rule also breaks the ordinality redistribution principle.

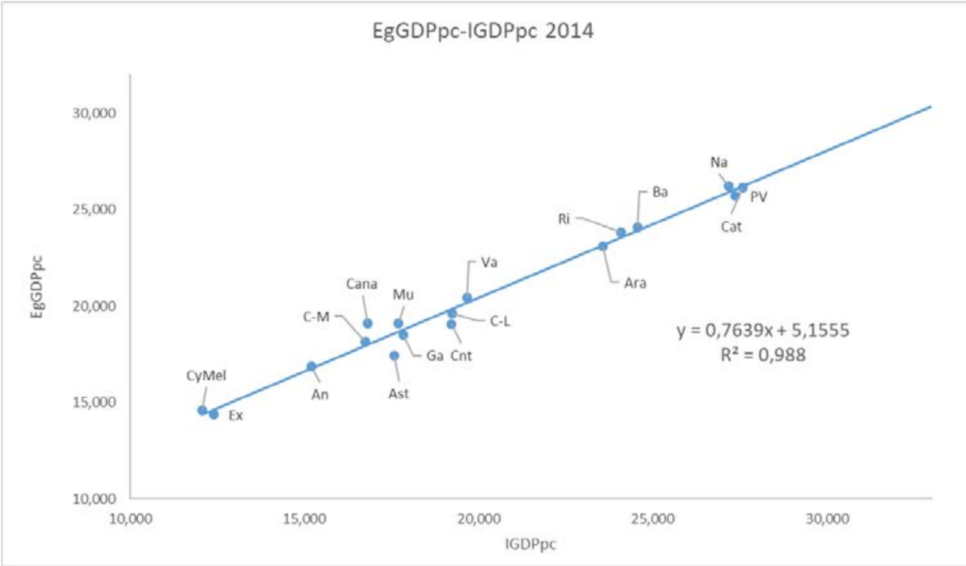

Figure 5

5 OPTIMISTIC VERSUS PESSIMISTIC CHARACTERISTIC FUNCTION

It can be argued that the characteristic function v_t (defined from a tax problem) is too *pessimistic* about how much tax a coalition of regions S can obtain when they withdraw from the system. In the definition of v_t we assume that all commercial relationships between regions S and $N \backslash S$ break down. However, it can be objected that, after a secessionist process, this extreme assumption is unrealistic, and we can expect that some trade activity would remain after the breakdown to a greater or less extent.

We recall that the purpose of the determination of the characteristic function v_t was twofold. On one hand, to build a plausible and computable reference point that can help us to evaluate, and compare between them, alternative ways of redistributing the total taxes generated in a country. Thus, it is closer to a normative rather than a descriptive approach. For each coalition of regions S, the amount $v_t(S)$ is the way of accounting for an *objective* reference point for such a coalition. Given two different tax rules χ and ψ, we can compare easily the dissatisfaction that both rules produce to coalition S by means of $v_t(S) - \chi(S)$ and $v_t(S) - \psi(S)$, respectively. On the other hand, the pessimistic approach in the determination of v_t guarantees that something out of $C(N, v_t)$ is undoubtedly unstable.

Obviously, towards a descriptive approach, we can complicate matters further by making more realistic breakdown assumptions. However, we should be cautious about the computational effect that an increment in the complexity of the determination of v_t could produce, as the number of coalitions, $2^n - 1$, increases exponentially in the size of n. Anyway, in what follows we offer a simple procedure to modify v_t. As part of its simplicity, it has the advantage that its computation remains in polynomial time.

It is clear that any hypothetical breakdown scenario that we can imagine, will restrict for sure the commercial relationships between the secessionist and the rest of the regions that remain in the country. This is the well-known *border effect*. It establishes that domestic agents trade more with each other than with foreign agents of the same size and distance. A classic case study is that of US-Canadian trade. In MacCallum (1995) and Helliwell (1996) it was shown that the interprovincial trade between Canadian provinces was more than 20 times larger than trade between Canadian provinces and American states in the period 1988-1990. This is a remarkable effect due to the fact that both states have low custom tariffs, that were being phased out by the 1988 Free Trade Agreement. In the European Union, Head and Mayer (2000) find that Europeans purchased 14 times more from domestic producers than from equally distant ones, for the average industry in 1985 (tariffs and quotas within the EU were phased out by 1968).

In a secessionist scenario, new borders appear with their corresponding custom tariffs. This has an economic impact because part of intra-national trade converts into international trade. The range of assumptions that can be made to determine the intensity in this effect can range from a totally friendly process to a much more traumatic one. For example, some cases of disintegration in former Eastern Bloc had been considered in Fidrmuc and Fidrmuc (2003). They find evidence of high level of economic integration before breakdown, with internal trade exceeding external trade intensity 24-fold (for Slovenia and Croatia) to 43-fold (the former Soviet Union and Czechoslovakia). Disintegration was followed by a sharp fall in trade intensity. After breakdown, these levels decrease to 2-fold in the case of Slovenia and Croatia, 7-fold for the former Czechoslovakia, 13-fold for the Baltics, and 30-fold for Belarus, Russia, and Ukraine.

We incorporate this fall in trade by a parameter α, $0 \leq \alpha \leq 1$. Suppose that a region $j \in N$ withdraws from the country N. Assume that for each region i within the country $N \backslash j$ the bilateral trade with j is affected in proportion α; hence now the coefficients will be αc_{ij} and αc_{ji}. Taxes collected will be affected in the same proportion. However, for commerce between countries, indirect taxes (value-added tax inside the EU, or custom tariffs from outside EU) are charged in the destination country. Therefore, indirect taxes I_{ij} associated to trade from region i to region j (collected initially in region i), now are collected as import tariffs in the new country j. For the same reason, indirect taxes I_{ji} associated to the exports from the new country j to region i (collected initially in region j), now are collected in region i as import tariffs. This change becomes particularly relevant when the secessionist region has a high positive net trade with the rest of the country.

Apart from the border effect, there are additional economic aspects in a secessionist process that are specific for the country in question. For example, Spain belongs to the European Union. This political fact implies that a region which becomes independent will be, at least temporally, out of the EU. Banks could be tempted to change their headquarters to another region in the country to guarantee the financial support of the European Central Bank. Well established companies in the Spanish and European market could also follow this offshoring process, fearful of losing their market share because of new (unknown) protective custom tariffs, or by nationalist boycott campaigns. Such considerations are of an uncertain nature and it is very difficult to assess in advance how the economic agents assess risks under this breakdown scenario. The addition of these negative aspects will decrease the expected value of α.

The analysis made for an individual region can be extended for any coalition of regions $S \subseteq N$ which wants to withdraw from the country. We assume that within S the intensity of the trade relationships does not change; hence they are able to manage $T(S)$, and, as a result of the border effect, the commerce with the remaining regions $N \backslash S$ is reduced in proportion α. This implies a reduction in the same percentage of direct taxes associated to the exports from S to $N \backslash S$ as well as indirect taxes associated to the imports from $N \backslash S$ to S. The parameter α reflects the optimistic/pessimistic perception of this breakdown process. Now the characteristic function v_t^α changes its definition accordingly.

It must be noticed that, up to now, when we have built the game (N, v_t), we have worked with the matrix T and submatrices of T. However, this will not be the case for v_t^α. Therefore, we explicitly assume in this Section that a tax problem is given by $(N; D, I) \in \mathcal{T}^N$, where N is the set of regions, D is the direct tax matrix as specified in (1), I is the indirect tax matrix as specified in (2), and the matrix of total taxes T is obtained from the sum of the direct and indirect tax matrices, $T = D + I$. Now we can define the tax game associated to the parameter α.

Definition 14 *Let a tax problem* $(N; D, I) \in \mathcal{T}^N$ *and coalition* $S \subseteq N$. *Given a parameter* α, *with* $0 \leq \alpha \leq 1$, *we define the tax game* v_t^α *by*

$$v_t^\alpha(S) = T(S) + \alpha \sum_{i \in S} \sum_{j \in N \backslash S} \left(D_{ij} + I_{ji}\right), \qquad \forall S \subseteq N. \tag{15}$$

The case $\alpha = 0$ corresponds to the most pessimistic perception, and is the original definition given in Section 1, that is $v_t^0 = v_t$. In such a case, payoffs x *outside the core* of the game, are clearly *unstable*. As it means that if $x \notin C(N, v_t^0)$, there exists a coalition S such that $v_t(S) > x(S)$. Therefore, the regions of S could improve the payoffs of x by cooperating together and dividing total taxes $v_t^0(S)$ among themselves, *even in the worst (most pessimistic) of possible breakdown scenarios*.
However, for payoffs $x \in C(N, v_t^0)$ we can have still doubts whether they can qualify as stable. This will depend on our perception of what is the true value for α. As, by definition, it holds that $v_t^\alpha(S) \leq v_t^{\alpha'}(S)$ for all $\alpha \leq \alpha'$, it follows that $C(N, v_t^{\alpha'}) \subseteq C(N, v_t^\alpha)$. If the optimistic perception about the economic possibilities of a coalition in case of secession increases, the set of stable payoffs is reduced.
What about the existence of stable allocations? We find again that the game v_t^α is convex, which implies that its core is non-empty.

Theorem 16 *The core* $C(N, v_t^\alpha)$ *is non-empty for every* $0 \leq \alpha \leq 1$.

Proof Let $i \in N$ and $S \subsetneq T \subseteq N \backslash \{i\}$. The marginal contribution in a tax game (N, v_t^α) is given by

$$m_i(S, v_t^\alpha) := v_t^\alpha(S \cup i) - v_t^\alpha(S) = t_{ii} + \alpha \sum_{j \in S} \left(D_{ij} + I_{ji}\right), \qquad \forall S \subseteq N \backslash \{i\}.$$

Thus, it is satisfied that

$$m_i(T, v_t) - m_i(S, v_t) = \alpha \sum_{j \in T \backslash S} \left(D_{ij} + I_{ji}\right) \geq 0,$$

and then (N, v_t^α) is convex. Therefore by Theorem 10 it holds that $C(N, v_t^\alpha) \neq \emptyset$. ■

Now we introduce a new tax rule, which computes how much tax a region can expect to collect in case of secession, for every value of α.

Definition 17 *For any tax problem* $(N; D, I) \in \mathcal{T}^N$ *and parameter* α, $0 \leq \alpha \leq 1$, *the secessionist tax rule* ζ^α *is defined by*

$$\zeta_i^\alpha(N; D, I) = t_{ii} + \alpha \sum_{j \in N \backslash i} \left(D_{ij} + I_{ji}\right), \forall i \in N.$$

The value ζ_i^α computes the individually rational payoffs of the tax game v_t^α; that is, $\zeta_i^\alpha(N; D, I) = v_t^\alpha(i)$. This tax rule ζ^α, in general is not an efficient rule; because, for $\alpha < 1$, it usually holds that

$$v_t^\alpha(N) - \zeta^\alpha(N) = (1 - \alpha) \sum_{i \in S} \sum_{j \in N \backslash S} \left(D_{ji} + I_{ij}\right) \neq 0.$$

Only when $\alpha = 1$ does efficiency hold.

For this fully optimistic case we call $\zeta = \zeta^1$ as the *optimistic secession rule*. We show now that ζ is always stable.

Proposition 18 *Let $(N; D, I) \in \mathcal{T}^N$ be a tax problem. Then it holds that $\zeta(N; D, I) \in C(N, v_t^\alpha)$, for all α, $0 \leq \alpha \leq 1$.*

Proof Denote by $\zeta = \zeta N; D, I)$. By construction $\zeta(N) = T(N)$, and then ζ satisfies efficiency. Let a coalition $S \subset N$ and $0 \leq \alpha \leq 1$, then we have that

$$\zeta(S) = T(S) + \sum_{i \in S} \sum_{j \in N \setminus S} \left(D_{ji} + I_{ij}\right) \geq T(S) + \alpha \sum_{i \in S} \sum_{j \in N \setminus S} \left(D_{ji} + I_{ij}\right) = v_t^\alpha(S)$$

hence $\zeta \in C(N, v_t^\alpha)$. ■

Moreover it is straightforward to check that when $\alpha = 1$, the allocation $\zeta(N; D, I)$ is the *unique* stable allocation in the game (N, v_t^1). This happens because v_t^1 is an additive characteristic function[18], and then $C(N, v_t^1) = \{\zeta\}$.

Notice that considering ζ as a tax rule is equivalent to considering all regions of the country as if they were a confederation of independent countries.

We illustrate these results with the help of Example 1. Suppose that the tax matrix is the sum of the following direct and indirect tax matrices:

$$T = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 4 & 3 \\ 1 & 3 & 5 \end{bmatrix}; \quad D = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 1 \\ 0 & 2 & 3 \end{bmatrix} + I = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 2 & 2 \\ 1 & 1 & 2 \end{bmatrix}.$$

The associated tax game (N, v_t^α) is given by

$$v_t^\alpha(1) = 2, v_t^\alpha(2) = 4, v_t^\alpha(3) = 5, v_t^\alpha(\{1,2\}) = 8 + \alpha(1+2),$$

$$v_t^\alpha(\{1,3\}) = 9 + \alpha(2+3), v_t^\alpha(\{2,3\}) = 15 + \alpha(1+1), v_t^\alpha(\{1,2,3\}) = 21.$$

The core of the game (N, v_t^α) is

$$C(N, v_t^\alpha) = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : 2 \leq x_1 \leq 6 - 2\alpha; 4 \leq x_2 \leq 12 - 5\alpha; 5 \leq x_3 \leq 13 - 3\alpha\}.$$

The optimistic secession tax rule is $\zeta(N; D, I) = (4, 7, 10)$.
In the next Figure 6 we draw the core of the tax game $(\mathrm{N}, \mathrm{v}_t^\alpha)$ for values $\alpha = 0, 0.5, 0.75, 1$.

[18] That is $v_t^1(S) = y(S)$ for all $S \subseteq N$.

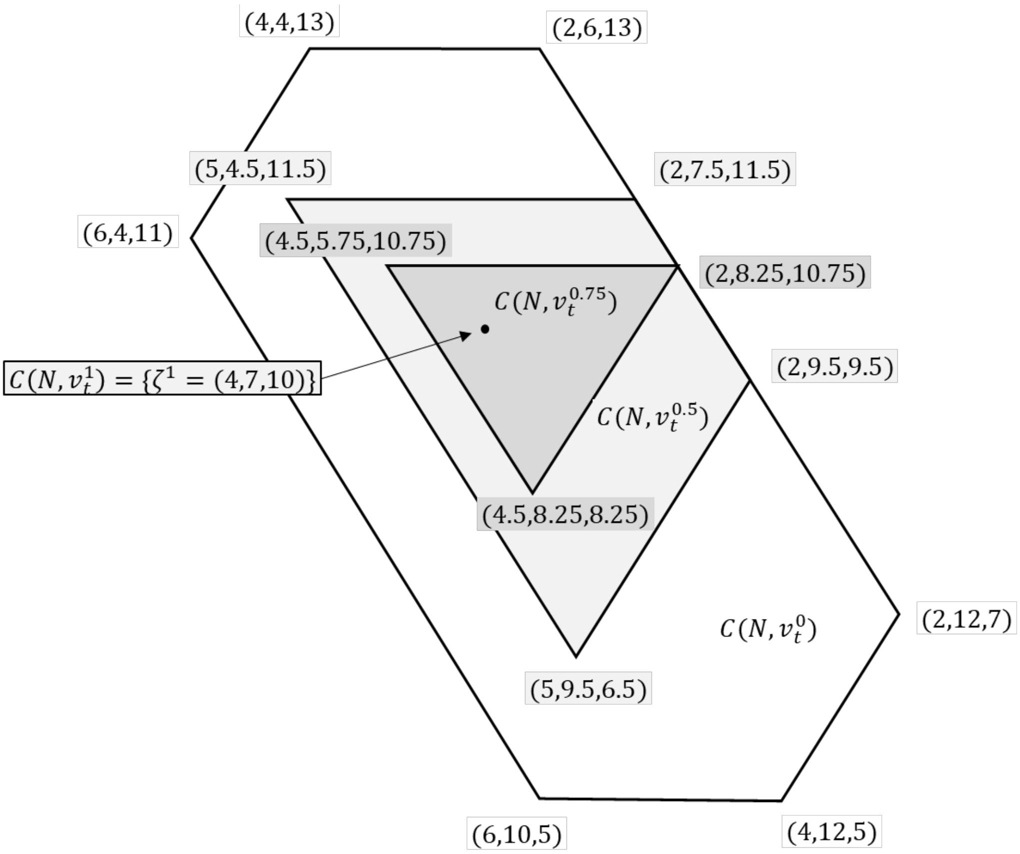

Figure 6

It could happen that a rich region with strong nationalist/secessionist ideology demands their tax redistribution to be equal to that which it could obtain with the optimistic secession rule. Any lower amount will only be considered as nothing less than robbery and plunder from the central government. If the budget obtained is lower than the threshold marked by ζ, this region could threaten to initiate a secession process in order to recover such loses inflicted by the state. Apart from the fact that such arguments ignore any kind of solidarity between regions, an obvious drawback of this consideration is the implicit assumption that this secessionist process is carried out under the most optimistic of possible scenarios, i.e. with $\alpha = 1$. However, this clearly contradicts the empirical evidence regarding the economic impact of the border effect. Indeed, it is equivalent to jumping out of the window and assuming that gravity will not apply.

We take again the Spanish case. There are two regions with high secessionist aspirations: The Basque Country (PV) and Catalonia (Cat). In the next Table 6 we compute the values of the secessionist rule ζ^α for several values of α and the differences in budget obtained with the present tax system *AE*.

		parameter α					
		1	0,4728219	0	1	0,4728219	0
AC's	*Adjusted Expenditures*	ζ^1	$\zeta^{0,47}$	$\zeta^0 = IR$	$AE - \zeta^1$	$AE - \zeta^{0,47}$	$AE - \zeta^0$
An	61.361.554	51.457.227	42.351.061	34.183.812	9.904.327	19.010.493	27.177.741
Ara	12.176.964	12.109.853	9.196.106	6.582.789	67.111	2.980.858	5.594.175
Ast	10.746.628	8.733.692	6.926.231	5.305.134	2.012.936	3.820.396	5.441.494
Ba	8.067.125	10.023.964	9.194.013	8.449.637	-1.956.840	-1.126.888	-382.512
Cana	16.514.603	12.767.252	10.370.827	8.221.492	3.747.351	6.143.776	8.293.111
Cnt	5.338.254	4.918.681	3.814.402	2.823.982	419.573	1.523.852	2.514.271
C-L	23.410.981	20.046.499	15.416.021	11.262.982	3.364.482	7.994.960	12.147.999
C-M	15.411.625	13.932.097	9.751.658	6.002.255	1.479.527	5.659.967	9.409.370
Cat	60.542.524	70.296.839	60.542.524	51.793.956	-9.754.315	0	8.748.568
Va	34.436.335	36.524.364	31.161.420	26.351.438	-2.088.029	3.274.915	8.084.897
Ex	9.491.444	6.694.142	5.179.746	3.821.497	2.797.302	4.311.698	5.669.948
Ga	23.932.122	19.642.107	16.789.166	14.230.385	4.290.015	7.142.956	9.701.737
Ma	48.949.930	66.861.289	54.973.610	44.311.644	-17.911.359	-6.023.680	4.638.286
Mu	9.880.471	10.106.533	8.040.012	6.186.565	-226.061	1.840.460	3.693.906
Na	5.847.499	5.811.665	4.614.462	3.540.699	35.833	1.233.037	2.306.799
PV	24.449.661	21.301.556	18.164.810	15.351.487	3.148.105	6.284.851	9.098.174
Ri	2.664.702	2.616.199	1.881.000	1.221.606	48.503	783.702	1.443.096
CyMel	1.752.879	1.131.340	750.476	408.881	621.538	1.002.403	1.343.998
Total	*374.975.300*	*374.975.300*	*309.117.545*	*250.050.242*	*0*	*65.857.755*	*124.925.058*

Table 6 Comparison of the Secession rule ζ and the Actual Spanish system *AE*

It is quite remarkable that the Basque Country (PV) is *in a better position with the present system (AE) than it would be under secession, even in the most optimistic case* (ζ^1). That is, under secession it would lose at least a budget of €3,148.1m. The case of Catalonia (Cat) is different. In the most optimistic case it will improve its budget by €9,754.3m. If we calculate the value of α^*_{Cat} such that Catalonia is indifferent to remaining in Spain or being independent, we obtain $\alpha^*_{Cat} = 0.4728$. It is clear that, from an economic point of view, the secessionist aspirations of Catalonia do not need to be supported by a very optimistic scenario. So it looks rational to claim a change of present regional financial status quo given by AE. Just the opposite is the case of Balearics (Ba). Even in the most pessimistic case, $\alpha = 0$, it should prefer always ζ^0, where obtaining its individually rational payoffs, it ends up in a better situation than it obtains under the present system AE. Note that the instability of the present Spanish rule AE is also expressed by the fact that for values greater than $\alpha^* = 0{,}2057$, the AC of Madrid (Ma) obtains a budget that is not even individually rational; i.e., $AE_{Ma} < \zeta^{\alpha^*}_{Ma}$. This implies that $AE \notin C(N, v^{\alpha}_t)$ for all $\alpha \geq \alpha^*$.

In Figure 7 we show that the overall redistributive effect of ζ^1 as a tax rule is the lowest of the rules considered up to now (AE, Eg and ζ^1), as the slope of the regression line is -0.0156. Accordingly, the slope of the regression line in Figure 8 is near 1, and its coefficient of dispersion R^2 is also near 1.

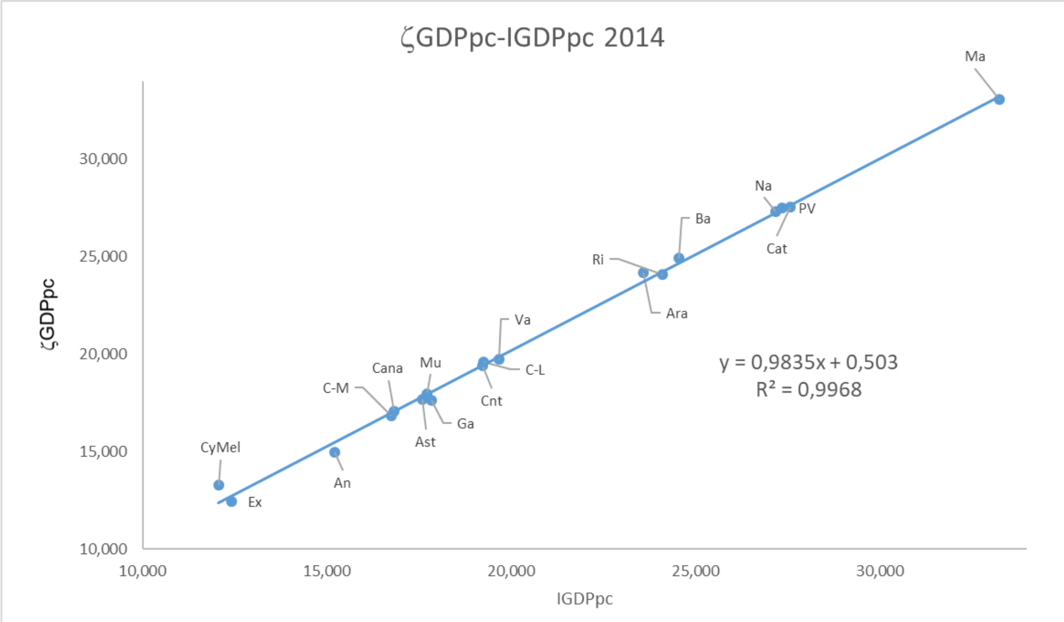

Figure 7

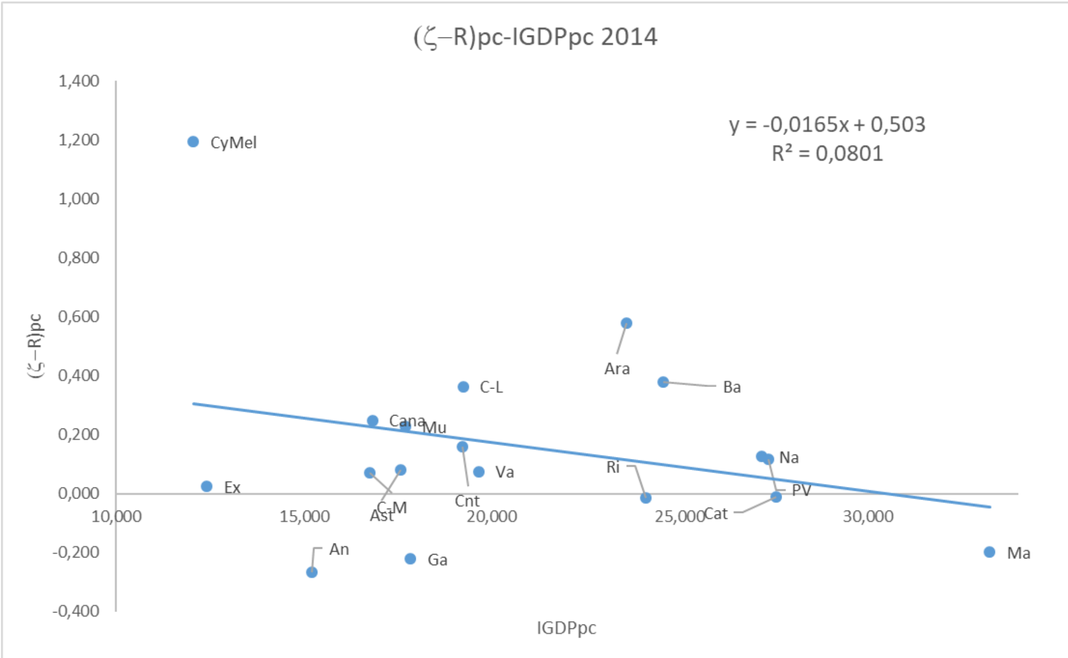

Figure 8

Table 7 shows the initial rank position of ACs in their per capita wealth and their final ranking after applying the present tax Spanish rule, the secessionist ζ^1 rule, and the population egalitarian rule Eg. The set of ACs which improve their per capita payoffs with ζ^1 respect to AE is $\{Ma, Ba, Cat, Va, Mu\}$. It is quite remarkable that this fact occurs not only for the rich regions of Madrid, Balearics and Catalonia, but it also occurs for Valencia and Murcia, which are two ACs with lower than the average per capita wealth.

Initial rank		Final rank					
AC's	*IGDPpc*	*AC's*	*AEGDPpc*	*AC's*	$\zeta^1 GDPpc$	*AC's*	*EgGDPpc*
Ma	33,247	Ma	30,240	Ma	33,049	Ma	30,635
Cat	27,565	PV	28,930	Cat	27,554	Na	26,239
PV	27,361	Na	27,361	PV	27,476	Cat	26,126
Na	27,179	Cat	26,236	Na	27,305	PV	25,710
Ba	24,564	Ri	24,233	Ba	24,942	Ba	24,067
Ri	24,095	Ara	24,202	Ara	24,151	Ri	23,821
Ara	23,574	Ba	23,195	Ri	24,079	Ara	23,106
Va	19,665	C-L	20,960	Va	19,739	Va	20,428
C-L	19,242	Cnt	20,097	C-L	19,606	C-L	19,612
Cnt	19,221	Ast	19,578	Cnt	19,382	Cana	19,114
Ga	17,846	Va	19,317	Mu	17,932	Mu	19,095
Mu	17,705	Ga	19,190	Ast	17,668	Cnt	19,063
Ast	17,589	Cana	18,838	Ga	17,624	Ga	18,525
Cana	16,821	Mu	17,777	Cana	17,069	C-M	18,158
C-M	16,754	C-M	17,540	C-M	16,825	Ast	17,454
An	15,211	CyMel	16,945	An	14,944	An	16,883
Ex	12,404	An	16,125	CyMel	13,260	CyMel	14,626
CyMel	12,066	Ex	14,986	Ex	12,429	Ex	14,380

Table 7 Initial and final ACs per capita wealth rank position

Finally, we wish to point out the cases of ‘Catalan Countries’ as a coalition: $CC = \{Cat, Ba, Va\}$, and the so called Foral Communities: $F = \{Na, PV\}$.
The so-called Catalan Countries are formed by (1) Catalonia, with high secessionist aspirations; (2) Balearics, with a well-founded grievance against the present tax rule AE, as its individual excess is negative; and (3) Valencia, whose $IGDPpc$ is lower than average, has a negative Adjusted Fiscal Balance, and ends, after the application of the AE tax rule, with a final $AEGDPpc$ even worse than initially. If we analyze the excess’ evolution of the coalition CC, from 2011 to 2014, in absolute terms it is decreasing, and its corresponding threshold values α^* are also decreasing, which implies that the dissatisfaction degree with respect to the status quo rises. If the present Spanish financial system does not change, the nationalist concept of ‘Catalan Countries’ could gain force in the future.

	2011	2012	2013	2014
$e(CC, AE, v^0)$	-14.388.768	-13.227.475	-9.346.161	-8.699.256
parameter α^*	0,546183258	0,555049823	0,429569041	0,386660433
$e(N\backslash F, AE, v^0)$	-10.770.732	-7.836.797	-6.491.299	-5.299.237
parameter α^*	0,928616777	0,839683045	0,7886116	0,624676117

Table 8 Foral Communities and Catalan Countries

The case of the Foral Communities $F = \{Na, PV\}$ is different. They are formed by the Basque Country (PV) and Navarre (Na). Both communities enjoy a different financial system, known as the *foral* system, from the rest of the Spanish ACs. Roughly speaking, except custom tariffs, both ACs collect almost all taxes inside their territory. A bilateral arrangement (*Concierto* signed with the Basque Country, and the *Convenio*, signed with Navarre) fix the tax transfer between each foral and Central Administration. Both communities have an $IGDPpc$ above average, and presently, are among the richer regions in Spain, and after the application of the AE tax rule, end up with a final $AEGDPpc$ even better than initially. Such a situation has been considered as a privilege with respect to the remaining regions that follow the common system. The tendency has been for such discriminatory positions to increase in recent years. If we

compute the excess obtained by Spain in the case where both foral communities withdraw from the country, i.e. the excess of coalition $N\backslash F$, its value is decreasing (in absolute terms) and accordingly the value of parameter α^* (which makes Spain indifferent to being with or without both foral communities) decreases.
It is desirable to find an agreement on a new regional financial system in Spain, improving its stability by minimizing the reasonable grievances that ACs can hold. Otherwise, this regional financial aspect will be source of increasing political instability.

6 STRATEGIC SUPPORT OF v_t^α

In this Section we offer an additional interpretation of the characteristic function v_t^α from a non-cooperative approach.

When regions bargain about the distribution of the total budget at their disposal, they wish to take into account the budget that each coalition S have at their disposal under secession. Such value is expressed by $v_t(S)$. When measuring such a value, we can consider that country N is split into two new countries, S and $N\backslash S$. This implies that the total taxes that S and $N\backslash S$ can collect depend on the trade relationships between them. The commercial trade c_{ij}, for $i \in S$ and $j \in N\backslash S$ depends on how much region j wishes to import from region i. Assume that coalition S can determine the intensity $\alpha_S \in [0,1]$ of how many goods regions i of S import from regions j of $N\backslash S$. And respectively, let $\alpha_{N\backslash S} \in [0,1]$ be the intensity of how many goods regions of $N\backslash S$ import from regions of S. Hence, the total direct taxes of S associated with the exports to $N\backslash S$ will depend on $\alpha_{N\backslash S}$, and the total indirect taxes of S due to the imports from $N\backslash S$ will depend on α_S. In summary, how much each coalition S can expect to collect depends on its own decision, α_S, and also on the decision of its complementary coalition, $\alpha_{N\backslash S}$.

Formally, for every pair of complementary coalitions, S and $N\backslash S$, the tax game is defined as

$$v_t[\alpha_S, \alpha_{N\backslash S}](S) = T(S) + \sum_{i\in S}\sum_{j\in N\backslash S}\left(\alpha_{N\backslash S} D_{ij} + \alpha_S I_{ji}\right),$$
$$v_t[\alpha_S, \alpha_{N\backslash S}](N\backslash S) = T(N\backslash S) + \sum_{j\in N\backslash S}\sum_{i\in S}\left(\alpha_S D_{ji} + \alpha_{N\backslash S} I_{ij}\right).$$

In the Literature, three different ways of deriving the characteristic function of a cooperative game have been considered: by (1) minimax values, (2) rational threats, and (3) non-cooperative equilibria[19]. We apply them to our setting.

(1) Von Neumann and Morgenstern (1944) suggested that $v(S)$ is the maximum sum of utility payoffs that the players of coalition S can guarantee themselves against the best offensive threat by the complementary coalition $N\backslash S$. Therefore, such a minimax representation of the tax game is defined by

$$\bar{v}_t(S) = min_{\alpha_{N\backslash S}} max_{\alpha_S} v_t[\alpha_S, \alpha_{N\backslash S}](S),$$

$$\bar{v}_t(N\backslash S) = min_{\alpha_S} max_{\alpha_{N\backslash S}} v_t[\alpha_S, \alpha_{N\backslash S}](N\backslash S).$$

[19] See Myerson (1991), Section 9.2, for a detailed discussion.

For any value $\alpha_{N\backslash S}$ the maximum payoff that coalition S can get is by making $\alpha_S = 1$. Hence, if coalition $N\backslash S$ wishes to minimize $v_t(S)$, its best strategy is to make $\bar{\alpha}_{N\backslash S} = 0$. The same reasoning applied to coalition $N\backslash S$ yields that best threat strategy of coalition S for minimizing the payoffs of $N\backslash S$ the value $\bar{\alpha}_S = 0$. Therefore, it holds that when both complementary coalitions apply their minmax strategies, we obtain the *pessimistic* tax game. That is,

$$\bar{v}_t(S) = v_t[\bar{\alpha}_S = 0, \bar{\alpha}_{N\backslash S} = 0](S) = v_t^0(S),$$

$$\bar{v}_t(N\backslash S) = v_t[\bar{\alpha}_S = 0, \bar{\alpha}_{N\backslash S} = 0](N\backslash S) = v_t^0(N\backslash S).$$

(2) Harsanyi (1963) proposed that characteristic functions can be obtained applying the *Nash' rational-threats* criterion (Nash 1953). Assume that two complementary coalitions, S and $N\backslash S$, decide to split equally the gains of their cooperation. Obviously, such gains will depend on what they would obtain in case they breakdown the cooperation. Denote by ϕ such cooperative payoffs. Therefore, the payoffs are

$$\phi[\alpha_S, \alpha_{N\backslash S}](S) = v_t[\alpha_S, \alpha_{N\backslash S}](S) + \frac{1}{2}[v_t(N) - v_t[\alpha_S, \alpha_{N\backslash S}](S) - v_t[\alpha_S, \alpha_{N\backslash S}](N\backslash S)],$$

$$\phi[\alpha_S, \alpha_{N\backslash S}](N\backslash S) = v_t[\alpha_S, \alpha_{N\backslash S}](N\backslash S) + \frac{1}{2}[v_t(N) - v_t[\alpha_S, \alpha_{N\backslash S}](S) - v_t[\alpha_S, \alpha_{N\backslash S}](N\backslash S)].$$

A pair $(\hat{\alpha}_S, \hat{\alpha}_{N\backslash S})$ of strategies will be mutually optimal rational-threats if

$$\phi[\hat{\alpha}_S, \hat{\alpha}_{N\backslash S}](S) \geq \phi[\alpha_S, \hat{\alpha}_{N\backslash S}](S), \quad \text{for all } \alpha_S, \text{ and}$$

$$\phi[\hat{\alpha}_S, \hat{\alpha}_{N\backslash S}](N\backslash S) \geq \phi[\hat{\alpha}_S, \alpha_{N\backslash S}](N\backslash S), \quad \text{for all } \alpha_{N\backslash S}.$$

From the necessary conditions of the optimality, it is easy to see that the above conditions are equivalent to

$$\hat{\alpha}_S \in \text{argmax}_{\alpha_S}[v_t[\alpha_S, \hat{\alpha}_{N\backslash S}](S) - v_t[\alpha_S, \hat{\alpha}_{N\backslash S}](N\backslash S)],$$

$$\hat{\alpha}_{N\backslash S} \in \text{argmax}_{\alpha_{N\backslash S}}[v_t[\hat{\alpha}_S, \alpha_{N\backslash S}](N\backslash S) - v_t[\hat{\alpha}_S, \alpha_{N\backslash S}](S)].$$

It follows that

$$\frac{\partial}{\partial \alpha_S}[v_t[\alpha_S, \alpha_{N\backslash S}](S) - v_t[\alpha_S, \alpha_{N\backslash S}](N\backslash S)] = \sum_{i\in S}\sum_{j\in N\backslash S} I_{ji} - \sum_{j\in N\backslash S}\sum_{i\in S} D_{ji}$$

$$\frac{\partial}{\partial \alpha_{N\backslash S}}[v_t[\alpha_S, \alpha_{N\backslash S}](N\backslash S) - v_t[\alpha_S, \alpha_{N\backslash S}](S)] = \sum_{j\in N\backslash S}\sum_{i\in S} I_{ij} - \sum_{i\in S}\sum_{j\in N\backslash S} D_{ij}$$

Indirect taxes are lower than direct. At least for the Spanish case, direct taxes collected are 2.96 times greater than indirect, as we can see in the next figure

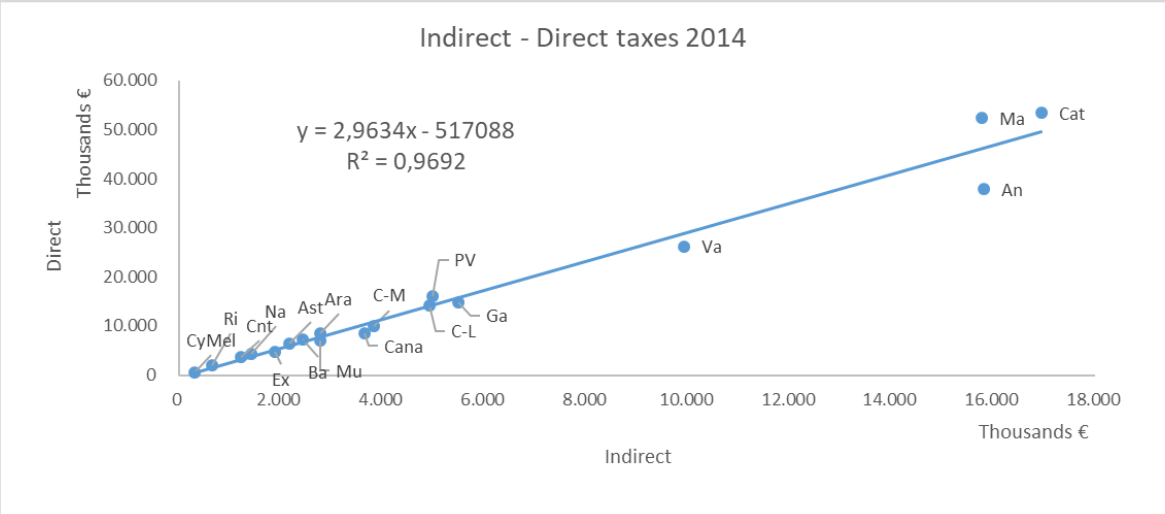

Figure 9

In summary, the derivatives are negative and hence, the optimality condition for maximization implies that $\hat{\alpha}_S = 0$ and $\hat{\alpha}_{N\backslash S} = 0$. Consequently, the characteristic function coincides with the pessimistic tax game, $\hat{v}_t \equiv v_t^0$.

(3) Finally, we can assume that complementary coalitions would play a pair of *equilibrium* strategies against each other. That is, $\left(\alpha_S^*, \alpha_{N\backslash S}^*\right)$ is a *Nash equilibrium* (Nash, 1951) if

$$\alpha_S^* \in \operatorname{argmax}_{\alpha_S} v_t\left[\alpha_S, \alpha_{N\backslash S}^*\right](S), \quad \text{and} \quad \alpha_{N\backslash S}^* \in \operatorname{argmax}_{\alpha_{N\backslash S}} v_t\left[\alpha_S^*, \alpha_{N\backslash S}\right].$$

By definition of $v_t\left[\alpha_S, \alpha_{N\backslash S}\right](S)$, a dominant strategy for coalition S is to make $\alpha_S = 1$, for any value $\alpha_{N\backslash S}$ given. Similarly, for coalition $N\backslash S$ its dominant strategy is also $\alpha_{N\backslash S} = 1$. Therefore, the Nash equilibrium is $\left(\alpha_S^* = 1, \alpha_{N\backslash S}^* = 1\right)$. We now obtain the opposite scenario: the characteristic function coincides with the optimistic tax game, $v_t^* \equiv v_t^1$.

Therefore, the pessimistic value $v_t^0(S)$ can be justified assuming either that $N\backslash S$ behave offensively against S, minimizing $\left(\bar{\alpha}_S = 0, \bar{\alpha}_{N\backslash S} = 0\right)$ the sum of taxes that S collect in case of secession, or that S and $N\backslash S$ play their rational threat strategies $\left(\hat{\alpha}_S = 0, \hat{\alpha}_{N\backslash S} = 0\right)$ when they negotiate over the possible division of $v_t(N)$. For the optimistic value $v_t^1(S)$, we can assume that in the unforeseen event of secession, coalition $N\backslash S$ will play $\alpha_{N\backslash S}$ *rationally*, i.e. maximizing the sum of payoffs of regions in $N\backslash S$ given α_S. Hence, in this breakdown event we are assuming that players behave rationally playing their Nash' equilibrium strategies $\left(\alpha_S^* = 1, \alpha_{N\backslash S}^* = 1\right)$. Such optimistic perception can also be justified if the secessionist process were an ultimatum game: first coalition S announces its secession with action α_S and second, coalition $N\backslash S$ decides which value of $\alpha_{N\backslash S}$ will play given the value α_S announced by S. As the best reaction function of $N\backslash S$ is to play $\alpha_{N\backslash S}(\alpha_S) = 1$, The best action of S is to play $\alpha_S = 1$ initially.

Up to now, we have assumed that every coalition S can choose freely any value α_S from 0 to 1. A more realistic perspective is to consider that external political and sociological factors will restrict the range in which the values α_S and $\alpha_{N\backslash S}$ can be decided into a more accurate interval $[\underline{\alpha}, \overline{\alpha}] \subset [0,1]$.

7 FINAL REMARKS

The purpose of this exercise is twofold: theoretical and applied. We wish to show the type of analysis that can be done with the help of the tax game. We believe it to be particularly relevant from the stability point of view in the debate regarding regional fiscal balances. The Spanish case has been used to illustrate how to compute the tax matrix $[T]$ and its associated tax game v_t. Obviously, the present analysis can be reproduced on every country with enough available data of taxes and inter-regional commercial trade.

As far as the tax game has been established, an immediate question that arises is the translation of well-known values defined in the setting of cooperative TU-games into the particular case of tax games. In the cooperative games theory literature, several single-valued solutions have been defined. Therefore, an obvious question that immediately arises is the translation of solutions defined in the setting of cooperative games into the setting of tax problems, by using its associated tax game. In the companion paper Calvo (2018), we show that three different solutions: the *Shapley value* (Shapley, 1953a,b), the *nucleolus* (Schmeidler, 1969), and the τ-value (Tijs, 1981), when they are applied to a tax game, all of them coincide in a tax rule that we call the *balanced tax* rule φ. As the tax game is convex by construction, the balanced tax rule is *stable.*

When there are wide differences in the per capita wealth among regions, the balanced tax rule φ usually exhibits a poor welfare redistribution behavior. For such reason, we also build a *p-balanced tax* rule φ^p, where the weights p are the per capita wealth of the regions. By its construction, this rule coincides with the weighted Shapley value (Shapley, 1953a) of the tax game and we show that, for the Spanish case, it offers a greater degree of solidarity among regions than φ.

The analysis of fiscal balances by means of the tax game tool relates only the ratio between the total expenditures made by all public administrations in the territory and the total revenues obtained. It can be used to decide *how much* should be spent on each territory, irrespective of *how* this amount will be distributed among the different public authorities. After deciding how much of the budget corresponds to each region, how to distributed this amount between the central and regional governments, in a more or less decentralized way, is indeed a different and separate, but not a trivial, problem to solve.

In case someone wishes to go further than a merely academic analysis, promoting a particular tax rule in a political scenario, it should be noted that a political agreement is needed, not only in the tax rule that has to be proposed, but also in the choice of how revenues and expenditures in each region should be computed. We tiptoe around this methodology aspect in the computation deliberately. In particular, we needed to specify whether we want to follow either the “tax-benefit incidence” or the “monetary flow” approaches (or something in between). Obviously, for practical purposes different methodologies[20] will yield different outputs for the same tax rule.

Finally, the present analysis can also be applied to a *confederation* of independent countries, as the European Union.

For example, suppose we wish to analyze the redistributive effect of a particular financial program in the EU. First, we need the total matrix T. Notice that indirect taxes to exports are charged in the destination country, hence the tax matrix must be redefined accordingly.

[20] A summary on this topic can be found in the IEB Report (2014).

The direct tax matrix D: For each country i of the confederation N, D_{ii} is the sum of all direct taxes associated to the internal economic activity, plus direct taxes associated to the exports outside the EU. $D_{ij}, i \neq j$, are direct taxes collected in i due to exports to $j \in N$.
The indirect tax matrix I: I_{ii} is the sum of all indirect taxes associated with the internal economic activity, plus custom tariffs associated to the imports from outside the EU. $I_{ij}, i \neq j$, are custom tariffs collected in j due to imports from $i \in N$. The total tax matrix is $T = D + I$. The total direct and indirect taxes collected in each country, denoted by γ, is given by

$$\gamma_i(N,T) = t_{ii} + \sum_{j \in N \backslash i} D_{ij} + \sum_{j \in N \backslash i} I_{ji}, \qquad \forall i \in N. \tag{19}$$

Its associated tax game (N, v_t) is defined in the same way: $v_t(S) = T(S)$ for all $S \subseteq N$. This game v_t is convex, its core $C(N, v_t)$ is non empty, and it is immediate to prove that $\gamma(N,T) \in C(N, v_t)$, because

$$e(S, \gamma, v_t) = -\sum_{i \in S} \sum_{j \in N \backslash S} (D_{ij} + I_{ji}) \leq 0, \qquad \forall S \subseteq N.$$

Now, given a financial program of total amount $\sum_{i \in N} f_i = 0$, where f_i is positive if country i is a receiver of funds, and negative when is a donor, the final effect of the program is $\gamma' = \gamma + f$. Similar stability analysis as made in the previous sections can also be done now for γ'.

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