

Entry with Two Correlated Signals

VNIVERSITAT
ID VALÈNCIA

>Alex Barrachina

Universitat de València and ERI-CES, Spain

>Yair Tauman

IDC Herzliya (Israel) and Stony Brook University, USA

>Amparo Urbano

Universitat de València and ERI-CES, Spain

November, 2014

Entry with Two Correlated Signals

Alex Barrachina*, Yair Tauman** and Amparo Urbano***

April 2014

Abstract

We analyze the effect of industrial espionage on limit-pricing models. We consider an incumbent monopolist engaged in R&D trying to reduce his cost of production and deter a potential entrant from entering the market. The R&D project may be successful or not and its outcome is a private information of the incumbent. The entrant has an access to an Intelligence System (IS hereafter) of a certain precision that generates a noisy signal on the outcome of the R&D project, and she decides whether to enter the market based on two signals: the price charged by the incumbent and the signal sent by the IS. It is assumed that the precision of the IS is exogenous and common knowledge. Our fundamental result is that for intermediate values of the IS precision, the set of pooling equilibria is non-empty even with profitable entry and the entrant enters if the IS tells her the R&D project was not successful. Since in the classical limit-pricing models the entrant never enters in a pooling equilibrium, the use of the IS by the entrant increases competition in pooling equilibrium with high probability. Moreover, the incumbent can deter profitable entry with positive probability.

JEL Classification: C72; D82; L10; L12

Keywords: Espionage; Entry deterrence; Asymmetric information; Pooling equilibria.

**Dept. of Economics, University Carlos III, Madrid (Spain). Alejandro.Barrachina@gmail.com*

***IDC Herzliya (Israel) and Stony Brook University (USA). Tauman@post.tau.ac.il*

****Dept. of Economic Analysis and ERI-CES, Univ of Valencia (Spain). Amparo.Urbano@uv.es*

Alex Barrachina and Amparo Urbano wish to thank the Ministry of Science and Technology and the European Feder Funds under projects ECO2010-20584, ECO2010-19596 and the Generalitat Valenciana under the Excellence Programs Prometeo 2009/068 and ISIC 2012/021 for their financial support.

1. Introduction.

Recent advances in communication and information technologies have increased firms' incentive to exceed the limits of competitive intelligence activities and led them to illegally and unethically acquire information about other firms. That is, to engage in *industrial espionage*¹. According to 1997 US State Department and Canadian Security and Intelligence Service Reports, industrial espionage costs US business over 8.16 billion dollar annually. Moreover, 43% of American firms have had at least six incidents of industrial espionage², embracing various activities in order to achieve cost advantages, to maintain market leadership, and the like. The issue is becoming so important that in February, 1st 2011 the *Financial Times* wrote "Taking into account all types of industrial espionage but counting only the cost to American businesses, US intelligence officials put the cost of lost sales due to illicit appropriation of technology and business ideas at \$100bn-\$250bn a year. General Motors, Ford, General Electric, Intel and Boeing are among the US companies known to have suffered from industrial espionage attacks, though all are wary of discussing the details".

In this paper we analyze the impact of industrial espionage on limit pricing and entry deterrence. The cost structure of an incumbent firm is very important information for a potential entrant contemplating market entry. Since this information is usually available in statements for internal use, the entrant firm could obtain it spying on the incumbent firm. The contribution of this paper is to extend the classical limit pricing model to the case where the potential entrant has an access to an intelligence system (IS, hereafter) to better detect the cost structure of an incumbent monopolist. Assuming that the precision of the IS is common knowledge, we show that spying on incumbent firms produces two types of results: 1) it increases competition with high probability and 2) there exist pooling equilibria where the incumbent can deter profitable entry with positive probability.

In a classic paper, Milgrom and Roberts (1982, MR hereafter) assume that the incumbent has private information about its costs of production and shows that a separating equilibrium may exist in which the incumbent limit prices and thereby

¹ Nasheri (2005).

² See Solan and Yariv (2004), footnote 1.

signals that its costs are low. The potential entrant then infers the incumbent's cost type and enters exactly when entry would be profitable under complete information. Pooling equilibria only exist when entry is not profitable and thus a striking implication is that profitable entry is not deterred. Bagwell and Ramey (1988) extend the model to allow the incumbent to have two signals: price and advertising³. In their model, the incumbent is privately informed as to whether its costs are high or low, the potential entrant's costs are commonly known, and entry is profitable if and only if the incumbent has high costs. In the refined separating equilibrium, the low cost incumbent engages in "cost-reducing distortion", in that it adopts the same price and advertising selection as it would were it, hypothetically, an uncontested monopoly with costs that were even lower. The low cost incumbent thus limit prices and distorts its demand-enhancing advertising upwards. Once again, due to signaling, profitable entry is not deterred. But, once pooling equilibria are considered Bagwell and Ramsey (1988) show that for some parameter regions refined pooling equilibria exist in which the high cost incumbent uses limit pricing and an upward distortion in advertising to deter entry that would be profitable under complete information. The MR result is in the benchmark model of Bagwell (2007), where both prices and advertising are signal for the incumbent monopolist. Bagwell (2007) extend the benchmark game to include two dimension of private information. Specifically, the incumbent is privately informed as to its cost type and its level of patience and selects price and advertising in the pre-entry period. He finds (intuitive) pooling equilibrium associated with the behavior of the patient high cost incumbent, which pools with the impatient low cost incumbent.

In this paper we deal with a monopoly who is engaged in R&D activity with the aim to reduce his cost of production. The outcome of the R&D project is the private information of the incumbent. A potential entrant assigns a certain probability that the monopolist fails to reduce his cost. If the project fails the entrant enter and obtains positive profit. Otherwise, if the project succeeds and the entrant enters, she will not be able to cover her entry cost and she will end up with negative profit. The entrant has

³ For other extensions, see Albaek and Overgaard (1992 a,b), Bagwell (1992), Bagwell and Ramey (1990, 1991), Harrington (1986,1987) and Linnemer (1998).

an access to an Intelligence System (IS) that allows her to gather (noisy) information about the incumbent's cost structure. The IS sends one out of two signals. The signal h , which indicates that the investment was not successful (in which case we refer to the incumbent as having the type H), and the signal l , which indicates that the investment was successful (namely, the incumbent is of type L).

It is assumed that the precision of the IS is exogenously given. This would be the case if the entrant firm has already a spying technology before she considers entering the market where the incumbent firm is operating (e.g. she has the ability to plant a Trojan horse in the computer system of the incumbent firm). The entrant decides whether to enter the market based on a pair of signals: the price that the incumbent charges for his product and the signal sent by the IS. If the entrant enters the market, she competes with the incumbent (whether it is a Cournot or Bertrand competition or any other mode of competition). It is assumed that the above is commonly known (including the precision of the IS).

The interaction between the entrant and the monopolist is described as a three stage game. In the first stage, the incumbent who knows the outcome of the R&D project, sets a price and the IS sends a signal. Based on this pair of signals the entrant decides whether to enter the market in the second stage. If she decides to enter, then she will be engaged in a certain mode of competition with the incumbent in the third stage of the game. The game is of incomplete information and, using Harsanyi's approach, we analyze it as a three player game, where the players are the two types of the incumbent and the entrant. We analyze the sequential equilibria of this game. The case where the IS precision is $\frac{1}{2}$ (not informative) is the limit pricing model of MR, but where the entrant's cost are common knowledge. We distinguish two cases: the first one is the separating equilibrium where the two incumbent types charge different prices and the second one is the pooling equilibrium case where both types charge the same price.

Several interesting findings emerge. Firstly, the entrant's best response entails two different threshold entry prices, one for each IS signal. That is, for each signal there is a threshold price such that the entrant enters if and only if the observed price is higher than the signal-related threshold price. Because of the correlation between signals and prices, these prices are ordered and thus the threshold price associated to signal l (the

incumbent is of type L) is higher than the one related to the other signal (signal h). This means that the entrant will stay out for a higher range of prices when observing l than when h is realized. Secondly, a group of findings offer support for predictions for separating equilibria that are featured in work by MR and Bagwell and Ramsey (1988): the low-cost incumbent separates from the high-cost type, and separation will be achieved through a cost-reducing distortion if the cost technology is not too far apart. In other words, at any separating equilibrium, the low-cost incumbent may limit prices; this behavior enables the potential entrant to infer the incumbent's cost so that profitable entry is not deterred when this type of incumbent exists. We show that the separating equilibria of our model coincide with that of (the modified) MR and the IS makes no difference for either the entrant or the incumbent. This is not very surprising since in a separating equilibrium the entrant identifies the incumbent's type with or without the use of the IS. The only difference between our separating equilibria and those of the aforementioned papers is in the behavior of the entrant for prices off the equilibrium path, for which she enters the market with positive probability. Thirdly, another group of findings redirects attention to pooling equilibria and provide support for the use of IS mechanism. A classical game-theoretical result is that limit pricing cannot deter profitable enter and thus the set of pooling equilibria when the entrant's expected profits are positive is empty. The same result is obtained in our model if the IS precision is sufficiently low to affect the entrant's decision. In the other extreme, if the IS precision is very accurate (close to 1), then contrary to the MR model, pooling equilibrium does not exist, even when entry is not profitable. In this case, the entrant identifies with high probability the incumbent's type and she will enter the market if the signal is the corresponding with the high type and she will stay out if the signal is the opposite one. The high cost monopolist, who knows that his type is detected with high probability, has an incentive to deviate to his monopoly price, upsetting a pooling equilibrium.

However, the results change for intermediate values of the IS precision, namely, when this precision is bounded away from 1 and from $\frac{1}{2}$. We show that the set of pooling equilibria is non-empty even under profitable entry and the monopoly price of the low cost monopoly is the highest pooling equilibrium price. The entrant's decision will be

still entering if the signal indicates that the investment was not successful and staying out if the signal indicates the opposite. When the IS precision is bounded away from 1, the high cost monopolist knows that with significant probability the entrant will obtain the wrong signal and will stay out. Hence, this type succeeds to fool the entrant about his type with positive probability.

To compare this result with the result obtained in the MR model, suppose first that prior to the completion of the R&D project, the expected payoff of E from entering the market is positive. Then, contrary to our model, no pooling equilibrium exists in the MR model because the entrant in a pooling equilibrium enters the market expecting positive profit and, hence, both types of monopolist are better off with their monopoly prices, upsetting the pooling equilibrium. Note that in the MR model the entrant never enters in a pooling equilibrium when the expected probability of entry is negative. Hence, the use of the IS increases competition in pooling equilibria with significant probability. On the other hand, our results also show that the incumbent can deter even profitable entry with high probability.

As already mentioned Bagwell and Ramey (1988) extend the MR model by allowing the incumbent to signal his costs with both price and advertisements. Hence, while in their paper both signals are sent by the incumbent, in our model the incumbent only signals his costs by the price and the other signal is generated by the IS operated by the entrant. Bagwell (2007) considers a more general game in which the incumbent has two dimensions of private information, his costs and his level of patience. He finds (intuitive) pooling equilibrium associated with the behavior of the patient high cost incumbent. In contrast, our model also offers pooling equilibria and profitable entry deterrence with positive probability, with only one dimension of private information by the incumbent but with two IS signals correlated with price, that provide additional (probabilistic) information about the incumbent type's. Surprisingly, the incumbent is better off with the use of a IS of intermediate precision by the entrant because signals help the entrant smooth her best response. The entrant's best response is completely smooth in Matthews and Mirman (1983), in a limit pricing model where demand is stochastic, so that prices reveal only statistical information about the incumbent's private information. Their (separating) equilibrium differs from standard signaling

equilibria in that it can be unique, it depends on prior beliefs and it is rich in comparative statics.

This paper is also related to Perea and Swinkels (1999) and Ho (2007, 2008) since they also consider espionage in the context of asymmetric information. However, in the present model the IS is not a decision maker who can act strategically as in Perea and Swinkels (1999) and Ho (2007, 2008). The work of Sakai (1985) analyzes two firms and information gathering in order to know the cost structure of the opponent firm. However, unlike us, the paper considers that both firms know neither the costs of their opponent nor their own costs. Finally, Barrachina *et al.* (2014) analyze the effect of industrial espionage on entry deterrence when the incumbent may expand capacity to deter entry.

The remainder of the paper is organized as follows. Section 2 sets out the model. Conditions for limit pricing are offered in Section 3, where Sub-section 3.1 presents the entrant's strategy and Sub-section 3.2 analyzes the separating equilibria of the game. Section 4 offers the pooling equilibria and Section 5 concludes the paper. Most of the proofs are presented in the Appendix.

2. The Model.

We consider a monopoly M and a potential entrant E. The monopoly M is engaged in R&D activity with the aim to reduce his cost of production from the current cost $C_H(q)$ to $C_L(q)$, where q is the production level. The outcome of the R&D project is the private information of M. The potential entrant, E, assigns a certain probability $\mu > 0$, that M fails to reduce his cost and probability $1 - \mu > 0$ that the project was successful. Therefore, the cost function of M is a private information and it can be of two types: L (low cost) and H (high cost) and the potential entrant, E, assigns probability μ that M is of type H. If the project fails and E enters, she obtains positive profit. Otherwise, if the project succeeds and E enters, she will not be able to cover her entry cost and she will end up with negative profit.

The entrant has an access to an Intelligence System (IS) that allows her to gather (noisy) information about the cost structure of M. The IS sends one out of two signals. The signal h , which indicates that the investment was not successful (in which case we

refer to M as having the type H), and the signal l , which indicates that the investment was successful (namely, M is of type L). The precision of the IS is α , $\frac{1}{2} \leq \alpha \leq 1$. That is, the signal sent by the IS is correct with probability α (for simplicity, whether the cost function is $C_H(q)$ or $C_L(q)$). The case where $\alpha = \frac{1}{2}$ is equivalent to the case where E does not use an IS. The case $\alpha = 1$ is the one where E knows exactly the outcome of the project. It is assumed that the precision α of the IS is exogenously given.

The interaction between E and M is described as a three stage game $G(\alpha)$. In the first period M chooses a price as a function of his type. The entrant decides whether to enter based on a pair of signals: the price, p that M charges for his product and the signal s (h or l) sent by the IS. If E enters, she will incur an entry cost K and compete with M (whether it is a Cournot or Bertrand competition or any other mode of competition). The form of competition (Cournot, Bertrand or other) is commonly known and once E enters, the outcome of the competition is assumed to be uniquely determined. It is assumed that the above is commonly known (including the precision α of the IS).

The game $G(\alpha)$ is a game of incomplete information and, using Harsanyi's approach, we analyze it as a three player game, where the players are the two types, H and L, of M and the entrant, E. The case where $\alpha = \frac{1}{2}$, namely, where the IS has no value (and, therefore, can be ignored), is exactly the limit pricing model MR, when the entrant only has an entry cost type. Therefore, our model is an extension of the MR model where the entrant has an access to an intelligence system and it is only for $\frac{1}{2} < \alpha < 1$.

Let $Q(p)$ be the demand function and $C_i(q)$ be the cost function of the t -type monopoly. Let D_H and D_L be the duopoly profits of the H-type and the L-type monopolists, respectively. For short we denote by H and L the H-type and the L-type monopolists, respectively. Let $\Pi_H(p)$ be the profit of H and let $\Pi_L(p)$ be the profit of L when they set the price p and when E does not enter. Denote by $D_E(H)$ and $D_E(L)$ the duopoly profits of E when she competes with H and L respectively. Denote

by p_H^M and p_L^M the monopoly prices of H and L respectively (and by q_H^M and q_L^M the monopoly quantities). The following assumptions are standard in the literature.

Assumptions

1. $D_E(L) - K \equiv \Delta_E(L) < 0$ and $D_E(H) - K \equiv \Delta_E(H) > 0$.
2. $\Pi_t(p)$, $t \in \{H, L\}$, is increasing in p whenever $p \leq p_t^M$ and is decreasing in p whenever $p \geq p_t^M$.
3. $\Pi_L(p_L^M) - D_L > \Pi_H(p_H^M) - D_H$. Namely, L loses from entry more than H.
4. The cost functions $C_t(x)$, $t \in \{H, L\}$, are differentiable, $C'_H(q) > C'_L(q)$ and $C_H(0) \geq C_L(0)$.
5. $Q(p)$ is differentiable and $Q'(p) < 0$ for all $p \geq 0$.
6. All the parameters of the model and the above five assumptions are commonly known.

Let \hat{p} be the price for H and let p_0 be the price for L that yields the duopoly profits for H and L respectively, i.e.,

$$\Pi_H(\hat{p}) = D_H \text{ and } \hat{p} < p_H^M$$

and

$$\Pi_L(p_0) = D_L \text{ and } p_0 < p_L^M.$$

Lemma 1. (i) $\Pi_L(p) - \Pi_H(p)$ decreases in p .

(ii) $p_H^M > p_L^M$.

(iii) $\hat{p} > p_0$.

Proof: See the Appendix.

We look for sequential equilibria of this game. A sequential equilibrium is an assessment of strategies and beliefs such that strategies are sequentially rational given the players' beliefs and beliefs are consistent in all information sets.

A strategy for firm E is an *entry rule*, $\sigma_E : \{h, l\} \times \mathbb{R} \rightarrow \{0, 1\}$. After observing a period one price $p \in \mathbb{R}_+$ and a signal $s \in \{h, l\}$, from the IS, firm E enters if $\sigma_E(s, p) = 1$ and

does not enter if $\sigma_E(s, p) = 0$. A strategy for firm M is a *pricing rule*, $p: \{H, L\} \rightarrow R$ that specifies a price, $p_t, t \in \{H, L\}$.

Given α , for every pair of signals (s, p) , $s \in \{h, l\}$ and $p \in \mathbb{R}_+$, let $Prob(H|s, p)$ and $Prob(L|s, p) = 1 - Prob(H|s, p)$ be the conditional probability that E assigns to the event that M is of type H and of type L, respectively.

It is assumed that, conditional on the type of M, the signals are mutually independent. Namely, M chooses the price p independently of the choice of the IS. Nevertheless, the signals p and s are correlated. If E observes a very high price, then it will be more likely to observe signal h . If however E observes a low price, it will be more likely to observe signal l . The Bayesian posterior belief that E assigns to the types of M is

$$\begin{aligned} Prob(H|h, p) &= \frac{Prob(h, p|H) Prob(H)}{Prob(h, p|H) Prob(H) + Prob(h, p|L) Prob(L)} \\ &= \frac{Prob(h|H) Prob(p|H) Prob(H)}{Prob(h|H) Prob(p|H) Prob(H) + Prob(h|L) Prob(p|L) Prob(L)} \end{aligned}$$

Equivalently,

$$Prob(H|h, p) = \frac{\mu \alpha f(p|H)}{\mu \alpha f(p|H) + (1 - \mu)(1 - \alpha) f(p|L)} \quad (1)$$

Similarly,

$$Prob(H|l, p) = \frac{\mu(1 - \alpha) f(p|H)}{\mu(1 - \alpha) f(p|H) + (1 - \mu)\alpha f(p|L)} \quad (2)$$

where $f(p|t)$ is the (density) probability that E assigns to the event that M of type t , $t \in \{H, L\}$ sends the signal p .

In a pure strategy equilibrium, if H assigns probability 1 to the event that $p = p_H$, then $f(p_H|H) = 1$ and $f(p|H) = 0$ if $p \neq p_H$. In this case, $f(p|H)$ is identified with the probability that H selects p . Similarly, $f(p_L|L) = 1$ and $f(p|L) = 0$, $\forall p \neq p_L$. Hence, for $p \neq p_H$ and $p \neq p_L$ (1) and (2) are not well defined for $p \notin \{p_H, p_L\}$ since the numerators and denominators are zero. To apply the sequential equilibrium concept we need to consistently define beliefs for any observed p , therefore off the equilibrium

path we approach $f(p|t)$ by a sequence $(f_n(p|t))_{n=1}^{\infty}$, such that $f_n(p|t) > 0$ and $\lim_{n \rightarrow \infty} f_n(p|t) = f(p|t)$ for all $p \in \mathbb{R}_+$. Let

$$Prob_n(H|h, p) \equiv \frac{\mu \alpha f_n(p|H)}{\mu \alpha f_n(p|H) + (1-\mu)(1-\alpha) f_n(p|L)} \quad (3)$$

$$Prob_n(H|l, p) \equiv \frac{\mu(1-\alpha) f_n(p|H)}{\mu(1-\alpha) f_n(p|H) + (1-\mu)\alpha f_n(p|L)} \quad (4)$$

Now $Prob_n(H|h, p)$ is well defined for all $p \in \mathbb{R}_+$ and (1) can be modified to be

$$Prob(H|h, p) \equiv \lim_{n \rightarrow \infty} \frac{\mu \alpha f_n(p|H)}{\mu \alpha f_n(p|H) + (1-\mu)(1-\alpha) f_n(p|L)}$$

We modify (2) in the same way. Note that different sequences of $(f_n(p|t))_{n=1}^{\infty}$ generate different conditional probabilities $Prob(t|s, p)$, $t \in \{H, L\}$, $s \in \{h, l\}$, $p \in \mathbb{R}_+$.

Let $\Pi_E(s, p)$ be the expected payoff of E given her on and off equilibrium beliefs, namely

$$\Pi_E(s, p) \equiv Prob(H|s, p)\Delta_E(H) + Prob(L|s, p)\Delta_E(L) \quad (5)$$

In a sequential equilibrium, if $\Pi_E(s, p) < 0$, E will not enter the market and if $\Pi_E(s, p) > 0$, E will enter. To simplify the analysis we assume that E stays out also when $\Pi_E(s, p) = 0$. Namely, E stays out if and only if she observes (s, p) such that

$$\Pi_E(s, p) \equiv Prob(H|s, p)\Delta_E(H) + Prob(L|s, p)\Delta_E(L) \leq 0$$

3. Conditions for limit pricing

3.1 The entry rule

For firm M to engage in limit pricing, entry should be more likely, in some sense, when prices are higher rather than lower for any observed signal. This is clearly the case if $\sigma_E(s, p)$ specifies entry if and only if for each signal s the observed price exceeds the entry price. The following assumptions will help insure that $\sigma_E(s, p)$ is of this form for any p and each $s \in \{h, l\}$.

Assumption 7.

(1) For each $t \in \{H, L\}$ and each n , $f_n(p|t)$ is differentiable in p for all $p \geq 0$.

(2) Let

$$g_n(p) = \frac{f_n(p|H)}{f_n(p|L)}$$

Then $g_n(p)$ is increasing in n for each p , and is increasing in p for each n .

Furthermore, for every n , $\lim_{p \rightarrow 0} g_n(p) = 0$ and $\lim_{p \rightarrow \infty} g_n(p) = \infty$.

(3) Let $g(p) = \lim_{n \rightarrow \infty} g_n(p)$. Then, $g(p)$ is continuous in p .

Notice that, $\frac{f(p|H)}{f(p|L)}$ is the likelihood ratio and to be increasing in p or, equivalently,

to satisfy the Monotone Likelihood Ratio Property in p (Milgrom, 1981) implies that a high price is more likely to come from the high cost monopoly H rather than from L.

Most common densities such as the uniform, normal or exponential satisfy the MLRP.

Assumption 7, guarantees continuity and monotonicity of the conditional probability

$Prob(t|s, p)$, $t \in \{H, L\}$, $s \in \{h, l\}$, $p \in \mathbb{R}_+$.

Lemma 2. (i) For each $s \in \{h, l\}$ and $t \in \{H, L\}$, $Prob(t|s, p)$ is continuous in p and

$Prob(H|s, p)$ is non-decreasing in p , $p \geq 0$.

(ii) For every $p \geq 0$, $Prob(H|h, p) > Prob(H|l, p)$.

Proof: (i) By (3),

$$Prob_n(H|h, p) = \frac{\mu\alpha \frac{f_n(p|H)}{f_n(p|L)}}{\mu\alpha \frac{f_n(p|H)}{f_n(p|L)} + (1-\mu)(1-\alpha)}$$

Hence,

$$Prob(H|h, p) = \frac{\mu\alpha g(p)}{\mu\alpha g(p) + (1-\mu)(1-\alpha)} \quad (6)$$

and by Assumption 7, $Prob(H|h, p)$ is continuous in p .

The proof that $Prob(H|l, p)$ is continuous is similarly derived by (4).

Since $Prob(L|s, p) = 1 - Prob(H|s, p)$, then $Prob(L|s, p)$ is also continuous.

Next note that $g(p)$ is non-decreasing in p since $g_n(p)$ is increasing in p for all n .

Therefore Theorem 1 in Milgrom (1981) implies that if $p_1 > p_2$ the posteriors

$Prob(H|s, p_1)$ dominates $Prob(H|s, p_2)$, $s = \{h, l\}$, in the sense of first order

stochastic dominance. In fact, it is easy to verify by (6) that $\frac{\partial}{\partial p} Prob(H|h, p) \geq 0$ if and

only if $g'(p) \geq 0$ and thus $Prob(H|h, p)$ is non-decreasing in p . The proof that

$Prob(H|l, p)$ is non-decreasing is similar.

(ii) Let

$$x_n(p) = \frac{Prob_n(H|h, p)}{Prob_n(H|l, p)}$$

By (3) and (4),

$$\begin{aligned} x_n(p) - 1 &= \frac{\frac{\alpha}{1-\alpha} [\mu(1-\alpha) f_n(p|H) + (1-\mu)\alpha f_n(p|L)]}{\mu\alpha f_n(p|H) + (1-\mu)(1-\alpha) f_n(p|L)} - 1 \\ &= \frac{(1-\mu) \frac{\alpha^2}{1-\alpha} f_n(p|L) - (1-\mu)(1-\alpha) f_n(p|L)}{\mu\alpha f_n(p|H) + (1-\mu)(1-\alpha) f_n(p|L)} \\ &= \frac{(1-\mu) f_n(p|L) (2\alpha - 1)}{(1-\alpha) [\mu\alpha f_n(p|H) + (1-\mu)(1-\alpha) f_n(p|L)]} \\ &= \frac{(1-\mu)(2\alpha - 1)}{(1-\alpha) [\mu\alpha g_n(p) + (1-\mu)(1-\alpha)]} \end{aligned}$$

Hence,

$$\lim_{n \rightarrow \infty} [x_n(p) - 1] = \frac{(1-\mu)(2\alpha - 1)}{(1-\alpha) [\mu\alpha g(p) + (1-\mu)(1-\alpha)]} > 0$$

Hence $\lim_{n \rightarrow \infty} x_n(p) > 1$ and, consequently, for every $p \geq 0$,

$$Prob(H|h, p) > Prob(H|l, p) \tag{7}$$

■

Lemma 3. Let $J_s = \{p \geq 0 \mid \Pi_E(s, p) \leq 0\}$. Then, J_s and $\mathbb{R}_+ \setminus J_s$ are both non-empty sets. In other words, $\Pi_E(s, p) < 0$ for sufficiently small p , and $\Pi_E(s, p) > 0$ for p sufficiently large.

Proof: By (5),

$$\begin{aligned} \Pi_E(s, p) &= \text{Prob}(H|s, p)\Delta_E(H) + \text{Prob}(L|s, p)\Delta_E(L) \\ &= \text{Prob}(L|s, p) \left[\frac{\text{Prob}(H|s, p)}{\text{Prob}(L|s, p)} \Delta_E(H) + \Delta_E(L) \right] \end{aligned} \quad (8)$$

Let $s = h$. For every p ,

$$\frac{\text{Prob}(H|h, p)}{\text{Prob}(L|h, p)} = \lim_{n \rightarrow \infty} \frac{\mu\alpha f_n(p|H)}{(1-\mu)(1-\alpha) f_n(p|L)} = \frac{\mu\alpha}{(1-\mu)(1-\alpha)} g(p)$$

We claim that $g(p) \rightarrow 0$ as $p \rightarrow 0$. This follows by Dini's theorem, as $g_n(p)$ is increasing in n , $g_n(p)$ is continuous in p and $g(p)$ is also continuous. Hence, for every $\delta > 0$, $\lim_{n \rightarrow \infty} g_n(p) = g(p)$ uniformly on $[0, \delta]$. Since for every n , $g_n(p) \rightarrow 0$ as $p \rightarrow 0$, we have $g(p) \rightarrow 0$ as $p \rightarrow 0$. Consequently,

$$\lim_p \frac{\text{Prob}(H|h, p)}{\text{Prob}(L|h, p)} = 0, \text{ as } p \rightarrow 0 \quad (9)$$

Inequality (9) holds also when h is replaced by l (the proof is similar).

Next, let us show that $\text{Prob}(L|h, p) > 0$ for small p .

$$\begin{aligned} \text{Prob}_n(L|h, p) &= \frac{(1-\mu)(1-\alpha) f_n(p|L)}{\mu\alpha f_n(p|H) + (1-\mu)(1-\alpha) f_n(p|L)} \\ &= \frac{1}{1 + \frac{\mu\alpha g_n(p)}{(1-\mu)(1-\alpha)}} \end{aligned} \quad (10)$$

Again, since $g_n(p) \rightarrow g(p)$ as $n \rightarrow \infty$ uniformly in any interval $[0, \delta]$, $\delta > 0$, and since $g(p) \rightarrow 0$ as $p \rightarrow 0$,

$$\text{Prob}(L|h, p) = \lim_n \text{Prob}_n(L|h, p) \rightarrow 1, \text{ as } p \rightarrow 0$$

In particular, $\text{Prob}(L|h, p) > 0$ for p sufficiently small. In a similar way, we can prove that $\text{Prob}(L|l, p) > 0$ for p sufficiently small.

Now, (8), (9) and the fact that $\Delta_E(L) < 0$ and $Prob(L|s, p) > 0$ for small p , imply that for sufficiently small p , $\Pi_E(s, p) < 0$ and $J_s \neq \emptyset$.

Let us show that for p sufficiently large, $\Pi_E(s, p) > 0$. We use the following claim.

Claim 1. $\lim_p Prob(L|s, p) = 0$ as $p \rightarrow \infty$.

Proof: Let $n = 1$ and $s = h$. By Assumption 7.2, $\lim \frac{f_1(p|H)}{f_1(p|L)} = \infty$. By (10),

$$Prob_1(L|h, p) \rightarrow 0 \text{ as } p \rightarrow \infty$$

Hence, for every $\varepsilon > 0$, there exists P s.t. for all $p > P$,

$$Prob_1(L|h, p) < \varepsilon$$

By (3),

$$Prob_n(H|h, p) = \frac{\mu\alpha}{\mu\alpha + (1-\mu)(1-\alpha) \frac{f_n(p|L)}{f_n(p|H)}}$$

By Assumption 7.2, $Prob_n(H|h, p)$ is increasing in n and, hence, $Prob_n(L|h, p)$ is decreasing in n for every p . Thus, for all $p > P$,

$$Prob_n(L|h, p) < Prob_1(L|h, p) < \varepsilon$$

Hence, for every $\varepsilon > 0$ and for all $p > P$,

$$Prob(L|h, p) = \lim_{n \rightarrow \infty} Prob_n(L|h, p) \leq \varepsilon$$

implying that

$$\lim_{p \rightarrow \infty} Prob(L|h, p) = 0$$

The proof that $Prob(L|l, p) = 0$, as $p \rightarrow \infty$ is similarly derived. ■

Claim 1 together with (5) imply that for p sufficiently large, $\Pi_E(s, p) > 0$, and the proof of Lemma 3 is completed. ■

Recall that by assumption 1, $D_E(L) - K \equiv \Delta_E(L) < 0$, $D_E(H) - K \equiv \Delta_E(H) > 0$. Then,

Lemma 4. Suppose that assumption 1 holds. Then, any beliefs of E which satisfy assumption 7, implies that $\Pi_E(s, p)$ is continuous and non-decreasing in p and

uniquely determine p_h and p_l , $p_h < p_l$, such that in every sequential equilibrium with these beliefs, E enters the market if and only if she observes the signal (h, p) with $p > p_h$ or the signal (l, p) with $p > p_l$.

Proof: By part (i) of Lemma 2 and by (5), $\Pi_E(s, p)$ is continuous and non-decreasing in p (this follows from the fact that $\text{Prob}(H|s, p)$ is continuous and non-decreasing in p , $\Delta_E(H) > 0$, $\text{Prob}(L|s, p) = 1 - \text{Prob}(H|s, p)$ and $\Delta_E(L) < 0$).

By Lemma 3, $\Pi_E(s, p) < 0$ for small p and $\Pi_E(s, p) > 0$ for sufficiently large p .

Let

$$p_h = \max \{ p \geq 0 \mid \Pi_E(h, p) \leq 0 \}$$

$$p_l = \max \{ p \geq 0 \mid \Pi_E(l, p) \leq 0 \}$$

By the continuity of $\Pi_E(s, p)$ in p ,

$$\Pi_E(h, p_h) = \Pi_E(l, p_l) = 0 \tag{11}$$

and E enters the market if and only if she observes either (h, p) s.t. $p > p_h$ or (l, p) s.t. $p > p_l$.

By (7) (part (ii) of Lemma 2) it is easy to verify that

$$\Pi_E(h, p) > \Pi_E(l, p) \tag{12}$$

By (11) and (12)

$$\Pi_E(l, p_l) = \Pi_E(h, p_h) > \Pi_E(l, p_h)$$

and since $\Pi_E(s, p)$ is non-decreasing in p , we have $p_l > p_h$. ■

The best response entry rule of E when she observes the pair of signals (s, p) is given by Figure 1 below.

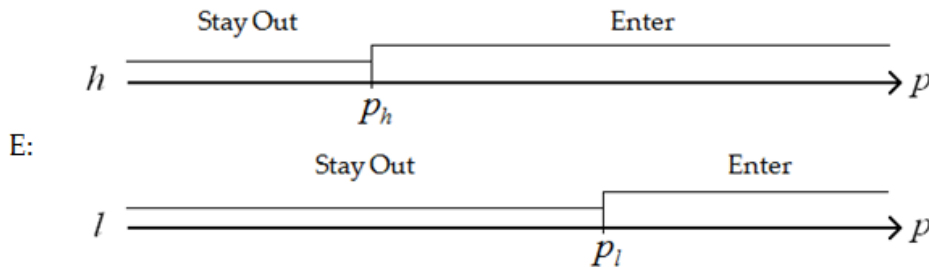


Figure 1

The correlation between signals and prices gives rise to the ordering of the threshold prices associated to signals and thus the threshold price associated to signal l is higher than the one related signal h . This means, for instance, that the entrant will stay out for a higher range of prices when observing l than when observing h . Our next goal is to characterize the sequential equilibrium of $G(\alpha)$ given the above decision rule of E.

3.2 Separating equilibria

A separating equilibrium is an incumbent's pair of prices $\{p_H, p_L$ with $p_H \neq p_L\}$, an entrant's entry rule, $\sigma_E(s, p)$ which are sequentially rational given beliefs $Prob(H|h, p)$ and $Prob(H|l, p)$, and beliefs are consistent with the players' strategies for any pair (s, p) . In this equilibrium E identifies with probability 1 the type of M. Hence, E enters the market when observing the price p_H irrespective of the signal of the IS, and E stays out when observing p_L , again irrespective of s . Therefore, by the entrant's strategy $p_H > p_l$ and $p_L \leq p_h$.

Some useful notation for the incentive compatibility constraints is the following. Let $\tilde{p}_t(\alpha)$ be the (unique) solution in p of the following equation,

$$\Pi_t(p) = \alpha \Pi_t(p_t^M) + (1-\alpha) D_t, \quad t \in \{H, L\}$$

where the expected profits of M in the second period are the monopoly profits with probability α and with probability $(1-\alpha)$ the duopoly profits. For instance if $t=L$, then

$$\Pi_L(p) = \alpha \Pi_L(p_L^M) + (1-\alpha) D_L,$$

thus, p is the price such that M's second period expected profits are a mixed combination of: if the entrant detects correctly the monopoly's type, she will stay out and if the entrant detects wrongly the monopoly's type, she will enter.

And let $\hat{p}_t(\alpha)$ be the unique solution in p of the following equation,

$$\Pi_t(p) = (1-\alpha) \Pi_t(p_t^M) + \alpha D_t,$$

where the expected profits of M in the second period are the monopoly profits with probability $(1-\alpha)$ and with probability α the duopoly profits. For instance if $t=H$

$$\Pi_H(p) = (1-\alpha) \Pi_H(p_H^M) + \alpha D_H,$$

Thus, p is the price such that M's second period expected profits are a mixed combination of: if the entrant detects wrongly the monopoly's type, she will stay out and if the entrant detects correctly the monopoly's type, she will enter.

The following proposition characterizes the sequential separating equilibrium prices of $G(\alpha)$.

Proposition 1. Consider the game $G(\alpha)$ for $\frac{1}{2} < \alpha < 1$, and let SSE be the set of all sequential separating equilibrium points of $G(\alpha)$. Let SSE_t be the set of all equilibrium prices of the t -type monopolist in SSE . Then, given consistent beliefs $Prob(H|h, p)$ and $Prob(H|l, p)$, for any p and s ,

$$(1) SSE_L = \left\{ p_L \mid p_0 \leq p_L \leq \min(p_L^M, \hat{p}) \right\} \text{ and } SSE_H = \{ p_H^M \}, \text{ and}$$

$$\sigma_E(s, p) = \begin{cases} 0, & s = h \text{ and } p \leq p_h, \\ 1, & s = h \text{ and } p > p_h, \\ 0, & s = l \text{ and } p \leq p_l, \\ 1, & s = l \text{ and } p > p_l. \end{cases}$$

$$(2) \text{ Let } p_L \in SSE_L. \text{ If } p_L < p_L^M, \text{ then } p_L = p_h. \text{ If } p_L = p_L^M, \text{ then } p_L^M \leq p_h.$$

Proof: (1) By lemma 4, given beliefs $Prob(H|h, p)$ and $Prob(H|l, p)$, for any p , $\sigma_E(s, p)$ maximizes the entrant's profits. Let us show that SSE_t maximizes the t -type monopoly's profits given $\sigma_E(s, p)$. The H-type monopoly, knowing that entry will occur is better off choosing the price p_H^M . Thus $SSE_H = \{ p_H^M \}$ and E enters for sure when she observes the price p_H^M . In particular, $p_H^M > p_l$.

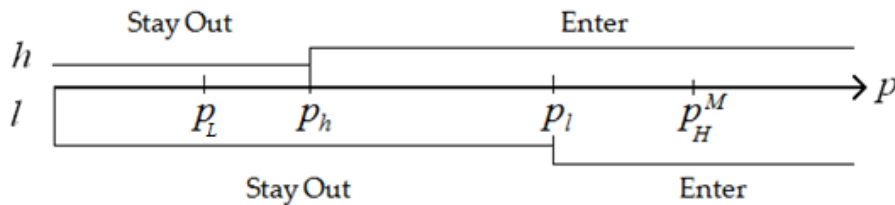


Figure 2

Next let us show that $SSE_L = \left\{ p_L \mid p_0 \leq p_L \leq \min(p_L^M, \hat{p}) \right\}$ for all α , $\frac{1}{2} < \alpha < 1$. We consider two cases. When the cost function of H is so significantly higher than that of L

that the duopoly price \hat{p} , when H competes with E, is above the monopoly price of L and when the cost functions are more similar.

Case 1: Suppose first that $p_L^M \leq \hat{p}$. We show that $p_L = p_L^M$ can be supported as a separating equilibrium price. Let p_h and p_l be such that

$$p_L = p_L^M \leq p_h \leq \hat{p} < p_l \leq \tilde{p}_H(\alpha) < p_H^M \quad (13)$$

To make sure that H has no incentive to deviate to either p_h or to p_l , the following two inequalities should hold

$$\Pi_H(p_H^M) + D_H \geq \Pi_H(p_h) + \Pi_H(p_H^M) \quad (14)$$

and

$$\Pi_H(p_H^M) + D_H \geq \Pi_H(p_l) + \alpha D_H + (1-\alpha)\Pi_H(p_H^M) \quad (15)$$

These two expressions are equivalent to

$$\begin{aligned} \Pi_H(\hat{p}) = D_H &\geq \Pi_H(p_h), \text{ and} \\ \Pi_H(p_l) &\leq \alpha\Pi_H(p_H^M) + (1-\alpha)D_H = \Pi_H(\tilde{p}_H(\alpha)). \end{aligned}$$

Thus $p_h \leq \hat{p}$ and $p_l \leq \tilde{p}_H(\alpha)$. By (13) the two incentive compatibility constraints of H are satisfied and hence $p_L^M \in SSE_L$

Next let p_L be such that $p_0 \leq p_L < p_L^M$. Let us show that we can support p_L as a separating equilibrium price. Let p_h and p_l be such that

$$p_L = p_h < p_l < p_L^M \leq \hat{p} < \tilde{p}_H(\alpha) < p_H^M \quad (16)$$

Similarly to the previous case, in order H has no incentive to deviate to either p_h or to p_l (14) and (15) must hold and thus $p_h \leq \hat{p}$ and $p_l \leq \tilde{p}_H(\alpha)$. Since $p_L^M \leq \hat{p} < \tilde{p}_H(\alpha)$ by (16) the two incentive compatibility constraints of H hold.

Next, since $p_L^M > p_l$, there are two relevant incentive compatibility constraints for L

$$\Pi_L(p_L) + \Pi_L(p_L^M) \geq \Pi_L(p_L^M) + D_L \quad (17)$$

and

$$\Pi_L(p_L) + \Pi_L(p_L^M) \geq \Pi_L(p_l) + \alpha\Pi_L(p_L^M) + (1-\alpha)D_L \quad (18)$$

(17) and (18) are equivalent to

$$\Pi_L(p_L) \geq D_L = \Pi_L(p_0) \quad (19)$$

$$\text{and} \quad \Pi_L(p_L) \geq \Pi_L(p_l) - (1-\alpha) \left[\Pi_L(p_L^M) - D_L \right] \quad (20)$$

Since $p_h = p_L \geq p_0$, (19) holds. As for (20), it holds for every $\alpha < 1$, provided that p_h is sufficiently close to p_l . Hence (16) guarantees that, for all $\frac{1}{2} < \alpha < 1$, $p_L \in SSE_L$ provided that $p_L = p_h$, p_l is sufficiently close to p_h and $p_l < p_L^M$.

Case 2: Suppose next that $\hat{p} < p_L^M$ and let p_L be such that $p_0 \leq p_L \leq \hat{p}$. We will show that for all α , $\frac{1}{2} < \alpha < 1$, $p_L \in SSE_L$. Let p_h and p_l be such that

$$p_L = p_h \leq \hat{p} < p_l < \min(p_L^M, \tilde{p}_H(\alpha)) < p_H^M \quad (21)$$

As in (14) and (15), the incentive compatibility constraints of H are equivalent to $p_h \leq \hat{p}$ and $p_l \leq \tilde{p}_H(\alpha)$. By (21) the two incentive compatibility constraints of H are satisfied.

Next, since $p_L^M > p_l$, in order for L not to deviate (19) and (20) must hold. Since $p_h = p_L \geq p_0$, (19) holds. Similar to Case 1, for every $\alpha < 1$, (20) holds if p_l is sufficiently close to p_h . Hence (21) guarantees that, for all $\frac{1}{2} < \alpha < 1$, $p_L \in SSE_L$ provided that $p_l - p_h$ is sufficiently small and $p_l < \min(p_L^M, \tilde{p}_H(\alpha))$.

Cases 1 and 2 prove that any price $p_L \in SSE_L$ if $p_0 \leq p_L \leq \min(p_L^M, \hat{p})$. Finally, we need to show that if $p_L \notin [p_0, \min(p_L^M, \hat{p})]$, then $p_L \notin SSE_L$. This proof is in the Appendix.

(2) We have to check that neither H nor L have incentives to deviate. In order to H not to deviate to p_L , the next inequality must hold

$$\Pi_H(p_H^M) + D_H \geq \Pi_H(p_L) + \Pi_H(p_H^M)$$

Equivalently,

$$\Pi_H(p_L) \leq D_H$$

or $p_L \leq \hat{p}$. Let us next check the incentive compatible constraint for L. In order to L not to deviate to p_L^M , the next inequality must hold

$$\Pi_L(p_L) + \Pi_L(p_L^M) \geq \Pi_L(p_L^M) + D_L$$

Equivalently,

$$\Pi_L(p_L) \geq D_L$$

or $p_L \geq p_0$. ■

Notice that in all separating equilibria $p_h \leq \hat{p}$ and $p_l \leq \tilde{p}_H(\alpha)$ must hold in order H does not deviate. Also notice that when the cost functions are not too far apart, i.e. $\hat{p} < p_L^M$, all the separating equilibria limit price: $p_0 \leq p_L < p_L^M$, while when the cost function of H is significantly higher than that of L in the sense that $p_L^M \leq \hat{p}$, then $p_L \leq p_L^M$. Therefore limit pricing is more likely in sequential separating equilibria when the cost technology is not too far apart, because in this case L needs a reduction of his monopoly price p_L^M in order to separate from H. When the cost function of H is significantly higher than that of L in the sense that $p_L^M \leq \hat{p}$, limit pricing will only occur when the monopoly price p_L^M is not too low in the sense that either $p_h < p_l < p_L^M$ or $p_h < p_L^M < p_l$.

A natural benchmark is when the IS precision is $\alpha = 1/2$ or, equivalently, a modification of the MR set up, when the entrant only has a cost type and does not operate an IS on M. This game is denoted by G_{MR} . Recall that \hat{p} is the price for H and that p_0 is the price for L that yields the duopoly profits for H and L respectively. In this game, the entrant's strategy $\sigma_E(p)$, is a threshold strategy,

$$\sigma_E(p) = \begin{cases} \text{"Stay out"}, & p \leq \bar{p} \\ \text{"Enter"}, & p > \bar{p} \end{cases}$$

where the threshold \bar{p} is the choice of E, given her beliefs

$Prob(H|p)$ and $Prob(L|p)$, for any p . This is easily derived if the likelihood ratio

$\frac{f(p|H)}{f(p|L)}$ is increasing in p or, equivalently, satisfies the Monotone Likelihood Ratio

Property in p (see Assumption 7). By a straightforward modification of lemma 4 the above entrant's strategy follows. Recall that $p_l > p_h$. Trivially, for $\alpha = 1/2$, $p_l = p_h = \bar{p}$.

Therefore, for any $\alpha > 1/2$, $\Pi_E(l, p_l) = \Pi_E(h, p_h) = \Pi_E(\bar{p}) > \Pi_E(l, p_h)$, that implies

$\Pi_E(l, p_l) > \Pi_E(l, p_h)$ and $\Pi_E(h, p_h) > \Pi_E(l, p_h)$ and hence $\bar{p} = p_h < p_l$.

Let p_H and p_L be the equilibrium strategies of H and L respectively. Proposition 2 below (proven in the Appendix) characterizes the separating equilibria in G_{MR} .

Proposition 2. Consider the game G_{MR} and let SSE^{MR} be the set of all sequential separating equilibrium points of G_{MR} . Then, given consistent beliefs $Prob(H|h, p)$ and $Prob(H|l, p)$, for any p ,

$$(1) SSE_L^{MR} = \left\{ p_L \mid p_0 \leq p_L \leq \min(p_L^M, \hat{p}) \right\} \text{ and } SSE_H^{MR} = \{ p_H^M \}, \text{ and}$$

$$\sigma_E(p) = \begin{cases} 0, & p \leq \bar{p}, \\ 1, & p > \bar{p}, \end{cases}$$

$$(2) \text{ Let } p_L \in SSE_L^{MR}. \text{ If } p_L < p_L^M, \text{ then } p_L = \bar{p}. \text{ If } p_L = p_L^M, \text{ then } p_L^M \leq \bar{p}.$$

Remark 1: By Lemma 1, $\hat{p} > p_0$ and SSE_{MR} is non-empty.

We wish to compare our equilibria in game $G(\alpha)$ with those of game G_{MR} . By the proofs of part (2) of the above of Propositions it is easily shown.

Corollary 1

- (1) The set SSE coincides with SSE_{MR} , the set of all sequential separating equilibrium points of G_{MR} .
- (2) Let $p_L \in SSE_L$ and suppose that $p_L < p_L^M$. Let p_h and \bar{p} be the equilibrium cutoff price for entry when in $G(\alpha)$ (when $s=h$) and in G_{MR} respectively. Then, $\bar{p} = p_h$.
- (3) Let $p_L \in SSE_L$ and suppose that $p_L < p_L^M$. Then the equilibrium strategy of E in $G(\alpha)$ coincides with the equilibrium strategy of E in G_{MR} for all $p_L \notin (p_h, p_l]$. If $p_L \in (p_h, p_l]$, then E in $G(\alpha)$ enters the market with positive probability (which is α if M is of type H and $1-\alpha$ if M is of type L) and enters for sure in G_{MR} .

Part (3) of the Corollary asserts that in $G(\alpha)$ E is less inclined to enter the market. For all prices below $\bar{p} = p_h$ E stays out of the market in both games G_{MR} and $G(\alpha)$. For prices above p_l , E enters for sure in both these games. But for prices p , $p_h < p \leq p_l$, E in $G(\alpha)$ enters the market if and only if the signal sent by the IS is h . In contrast, E in

this region enters the market for sure in the game G_{MR} . The difference between $G(\alpha)$ and G_{MR} with regard to sequential separating equilibria is only in the behavior of E off the equilibrium path. Therefore, for prices off the equilibrium path the monopolist is better off with an entrant with access to an IS of commonly known precision.

4. Conditions for entry deterrence

At any sequential separating equilibria the probability of entry for any s is the probability of H, as in the complete information case, except for prices p off the equilibrium path belonging to the interval $(p_h, p_l]$, where E will enter the market with probability $\alpha > 0$ if she receives signal h and $(1-\alpha) > 0$ if she receives signal l .

We analyze next, the sequential pooling equilibria. The received literature showed that profitable entry is never deterred by the incumbent monopolist. We claim here that the incumbent monopolist can deter profitable entry if either the cost function of H is not significantly higher than that of L or the IS precision belong to some intermediate levels.

Pooling Equilibrium.

By pooling equilibrium we refer to triples of the form (σ_E, p_H, p_L) where σ_E is the strategy of E and $p_H = p_L \equiv p^*$.

Given the signal l of the IS, the expected payoff of E conditional to receiving signal l is

$$\Pi_E(l|\alpha) \equiv Prob(H|l)\Delta_E(H) + Prob(L|l)\Delta_E(L)$$

Equivalently,

$$\Pi_E(l|\alpha) = \frac{\mu(1-\alpha)}{\mu(1-\alpha) + (1-\mu)\alpha} \Delta_E(H) + \frac{(1-\mu)\alpha}{\mu(1-\alpha) + (1-\mu)\alpha} \Delta_E(L)$$

Hence, if the IS sends the signal l , E does not enter the market when observing the price p^* if and only if

$$\Pi_E(l|\alpha) \leq 0 \tag{22}$$

Let

$$\bar{\alpha}_l = \frac{\mu\Delta_E(H)}{\mu\Delta_E(H) - (1-\mu)\Delta_E(L)}$$

Note that (22) holds and E does not enter if and only if $\alpha \geq \bar{\alpha}_l$.

Notice that $\mu\Delta_E(H) + (1-\mu)\Delta_E(L)$ is the entrant's expected profit without the IS.

Since $\Delta_E(L) < 0$, $0 < \bar{\alpha}_l < 1$ then $\bar{\alpha}_l < \frac{1}{2}$ if and only if

$$\mu\Delta_E(H) + (1-\mu)\Delta_E(L) < 0 \quad (23)$$

Therefore for $\frac{1}{2} < \alpha < 1$ E does not enter if and only if (23) holds.

Suppose next that the IS sends the signal h . Then the expected payoff of E conditional to receiving h is

$$\Pi_E(h|\alpha) \equiv \text{Prob}(H|h)\Delta_E(H) + \text{Prob}(L|h)\Delta_E(L)$$

Equivalently,

$$\Pi_E(h|\alpha) = \frac{\mu\alpha}{\mu\alpha + (1-\mu)(1-\alpha)}\Delta_E(H) + \frac{(1-\mu)(1-\alpha)}{\mu\alpha + (1-\mu)(1-\alpha)}\Delta_E(L)$$

Hence, if the IS sends the signal h , E does not enter the market when observing the price p^* if and only if $\Pi_E(h|\alpha) \leq 0$.

Let

$$\bar{\alpha}_h = \frac{-(1-\mu)\Delta_E(L)}{\mu\Delta_E(H) - (1-\mu)\Delta_E(L)} \quad (24)$$

Note that $\Pi_E(h|\alpha) \leq 0$ if and only if $\alpha \leq \bar{\alpha}_h$.

Since $\Delta_E(L) < 0$, $0 < \bar{\alpha}_h < 1$. Note that $\bar{\alpha}_h > \frac{1}{2}$ if and only if (23) holds, i.e. the entrant's expected profit without signals is negative.

Corollary 2. Suppose that $\frac{1}{2} < \alpha < 1$ and

$$\mu\Delta_E(H) + (1-\mu)\Delta_E(L) < 0$$

Then E stays out if and only if she observes the signal l or if $\alpha \leq \bar{\alpha}_h$.

In other words, if (23) holds, the entrant enters the market if and only if the signal is h and $\alpha > \bar{\alpha}_h$. Recall that p_H^M and p_L^M are the monopoly prices of H and L respectively and that \hat{p} is the monopoly price of H that yields the duopoly profits. Let δ be the

value of α such that the incumbent H does not deviate from a pooling equilibrium p_L^M to p_H^M , when $p_L^M > \hat{p}$, i.e.

$$\Pi_H(p_L^M) + \alpha D_H + (1 - \alpha)\Pi_H(p_H^M) \geq \Pi_H(p_H^M) + D_H$$

or

$$\delta = \frac{\Pi_H(p_L^M) - D_H}{\Pi_H(p_H^M) - D_H} = \frac{\Pi_H(p_L^M) - \Pi_H(\hat{p})}{\Pi_H(p_H^M) - \Pi_H(\hat{p})} \quad (25)$$

Clearly $0 < \delta < 1$.

The following proposition characterizes the pooling equilibria of the game $G(\alpha)$.

Proposition 3. Consider the game $G(\alpha)$, where $\frac{1}{2} < \alpha < 1$. Let $SPEP$ be the set of all sequential pooling equilibrium prices and SPE the set of all sequential pooling equilibria of $G(\alpha)$.

(1) Suppose that expected profits (not conditioned to the IS signals) from entry are negative, i.e., $\mu\Delta_E(H) + (1 - \mu)\Delta_E(L) < 0$. Then

- (i) If $p_L^M < \hat{p}$ (the cost technology is quite far apart), then $SPE = \emptyset$,
- (ii) If $p_L^M = \hat{p}$ and $\alpha \leq \bar{\alpha}_h$, then $SPEP = \{p_L^M\}$. If $\alpha > \bar{\alpha}_h$, then $SPE = \emptyset$.
- (iii) If $p_L^M > \hat{p}$ (intermediate cost technology) then
 - (iii.1) For $\alpha \leq \bar{\alpha}_h$, in every equilibrium in SPE , E stays out irrespective of the signal s and $SPEP = [\hat{p}, p_L^M]$.
 - (iii.2) If $\bar{\alpha}_h < \delta$, then for all α , $\bar{\alpha}_h < \alpha \leq \delta$, E enters if and only if $s = h$, $SPE \neq \emptyset$ and $SPEP = [\max(\tilde{p}_H(\alpha), \hat{p}_L(\alpha)), p_L^M]$.
 - (iii.3) For $\alpha > \delta$, $SPE = \emptyset$.

(2) Suppose that expected profits (not conditioned to the IS signals) from entry are positive, i.e. $\mu\Delta_E(H) + (1 - \mu)\Delta_E(L) > 0$. Then,

- (i) If $p_L^M \leq \hat{p}$ (the cost technology is quite far apart), then $SPE = \emptyset$,
- (ii) If $p_L^M > \hat{p}$ (intermediate cost technology) ten,
 - (ii.1) For $\alpha < \bar{\alpha}_l$, $SPE = \emptyset$.

(ii.2) If $\bar{\alpha}_l \leq \delta$, then for all α , $\bar{\alpha}_l \leq \alpha \leq \delta$, E enters if and only if $s = h$,
 $SPE \neq \emptyset$ and $SPEP = \left[\max(\tilde{p}_H(\alpha), \hat{p}_L(\alpha)), p_L^M \right]$.

(ii.3) For $\alpha > \delta$, $SPE = \emptyset$.

(3) Suppose that $\delta < \max(\bar{\alpha}_l, \bar{\alpha}_h) = \bar{\alpha}_l$, then $SPE = \emptyset$. Suppose that
 $\delta < \max(\bar{\alpha}_l, \bar{\alpha}_h) = \bar{\alpha}_h$, then $SPE = \emptyset$ whenever⁴ $\alpha > \bar{\alpha}_h$.

Proof: See the Appendix.

Proposition 3 asserts that sequential pooling equilibrium does not exist if either $p_L^M < \hat{p}$ or if $\alpha > \delta$. The first condition, $p_L^M < \hat{p}$, implies that the cost function of H is significantly higher than that of L. Even the duopoly price \hat{p} , when H competes with E, is above the monopoly price of L. In this case, it is too costly for H to mimic L and to fool E about his type. The other condition, $\alpha > \delta$, means that the IS is sufficiently accurate so that when E observes the signal h , she knows that the true type of M is H with high probability, and she is better off entering the market. In this case, H, who knows that his type is detected with high probability, has no reason to pool and he is better off charging the monopoly price p_H^M , upsetting the pooling equilibrium.

For intermediate values of α ($\bar{\alpha}_h < \alpha \leq \delta$ or $\bar{\alpha}_l \leq \alpha \leq \delta$), the set of pooling equilibria is non-empty even under profitable entry and the decision of E is to enter the market if and only if the signal sent by the IS is h . In this case, M of type H knows that α is sufficiently low so with significant probability, $(1 - \alpha)$, E will obtain the wrong signal l and will stay out. However, it is also required that the precision α is not too low since, otherwise, E will not trust the signal and she will enter whether the signal is h or l . But then, the two type monopolists will be better off with their monopoly prices, upsetting a pooling equilibrium.

Note that when $\alpha = \delta$, then $\max(\tilde{p}_H(\alpha), \hat{p}_L(\alpha)) = \tilde{p}_H(\alpha) = p_L^M$ and $SPEP = \{p_L^M\}$.

Proposition 3 also asserts that if $\mu \Delta_E(H) + (1 - \mu) \Delta_E(L) > 0$ (in which case $\bar{\alpha}_h < \bar{\alpha}_l$) and if α is relatively small ($\alpha < \bar{\alpha}_l$), then $SPE = \emptyset$. In other words, pooling equilibrium does not exist since E will enter the market and both types of M are better

⁴ It is easy to verify that if $\delta = \bar{\alpha}_h$, then $SPE = \emptyset$ whenever $\alpha > \bar{\alpha}_h$.

off deviating to their monopoly price. In contrast, when $\mu\Delta_E(H) + (1-\mu)\Delta_E(L) < 0$ and if α is relatively small $\alpha \leq \bar{\alpha}_h$, then $SPE \neq \emptyset$, but E stays out irrespective of the signal s . Hence, the use of a relatively not accurate IS has no impact on either entry or on entry deterrence, for relatively small α .

Nevertheless, the incumbent can deter profitable entry with significant probability, for intermediate values of the IS precision. Namely, for $\bar{\alpha}_l \leq \alpha \leq \delta$, $\mu\Delta_E(H) + (1-\mu)\Delta_E(L) > 0$, entry will be deterred if the signal sent by the IS is l . This probability is $\alpha > 1/2$ when M is of type L and $(1-\alpha)$ when M is of type H.

Remark 2. The relationship between δ and $\bar{\alpha}_h$ or $\bar{\alpha}_l$ in game $G(\alpha)$ is not obvious and in general it is quite complex.⁵

When $\alpha = 1/2$, then game $G(\alpha)$ collapses to G_{MR} and the entrant's expected profit is now $\mu\Delta_E(H) + (1-\mu)\Delta_E(L)$. We offer the pooling equilibria of G_{MR} , whose proof is in the Appendix.

Proposition 4. Consider the game G_{MR} . Let $SPEP_{MR}$ be the set of all sequential pooling equilibrium prices and SPE_{MR} the set of all sequential pooling equilibria of G_{MR} . Then,

(1) When $\mu\Delta_E(H) + (1-\mu)\Delta_E(L) < 0$

$$(i) \quad SPEP_{MR} = \{p_H = p_L = p^* = \bar{p}\}$$

and

$$(ii) \quad \hat{p} \leq p^* \leq p_L^M$$

(2) When $\mu\Delta_E(H) + (1-\mu)\Delta_E(L) > 0$, the set of all sequential pooling equilibria

in G_{MR} is the empty set: $SPE_{MR} = \emptyset$

Notice that the set of sequential pooling equilibria of G_{MR} will be not empty if $\hat{p} \leq p_L^M$.

⁵ In light of part (3) of Proposition 3 it would be important to shed a light on this relationship. The author analyze this relationship for the linear demand and linear cost functions case, assuming a Cournot competition if E enters the market. They show that that if α is not very accurate and the demand is not too small, then $\delta > \max(\bar{\alpha}_l, \bar{\alpha}_h)$ and p_L^M is a sequential pooling equilibrium price. In particular $SPEP \neq \emptyset$. This analysis is available upon request.

A comparison of Proposition 3 and 4 shows that:

Corollary 3.

- (1) Suppose that $p_L^M > \hat{p}$ (intermediate cost technology) and expected profits (not conditioned to the IS signals) from entry are negative, i.e., $\mu\Delta_E(H) + (1-\mu)\Delta_E(L) < 0$. Then, for $\alpha \leq \bar{\alpha}_h$, the set *SPEP* in $G(\alpha)$ is non-empty and coincides with the set $SPEP_{MR}$ of all sequential pooling equilibrium prices of the game G_{MR} .
- (3) Suppose that $p_L^M > \hat{p}$ (intermediate cost technology) and expected profits (not conditioned to the IS signals) from entry are positive, i.e., $\mu\Delta_E(H) + (1-\mu)\Delta_E(L) > 0$. Then, for $\alpha < \bar{\alpha}_l$ and $\alpha > \delta$, the set of all sequential pooling equilibria in both $G(\alpha)$ and G_{MR} is the empty set. For $\bar{\alpha}_l \leq \alpha \leq \delta$, *SPEP* in $G(\alpha)$ is non-empty.

Proposition 4 shows that when $\mu\Delta_E(H) + (1-\mu)\Delta_E(L) > 0$, the entrant will enter in G_{MR} when observes price p^* . Hence both types H and L of monopoly should select their monopoly prices p_H^M and p_L^M , respectively, destroying the pooling equilibria. Therefore, profitable entry is never deterred. This result is maintained even when the incumbent monopolist does not know the entry costs of the entrant (see MR) and in the benchmark model of Bagwell and Ramey (1988), where both prices and advertising are signal for the incumbent monopolist. Bagwell (2007) extend the benchmark game to include two dimension of private information. Specifically, the incumbent is privately informed as to its cost type *and its level of patience* and selects price and advertising in the pre-entry period. He finds (intuitive) pooling equilibrium associated with the behavior of the patient high cost incumbent, who pools with the impatient low cost incumbent.

In contrast, our model (Proposition 3) also offers pooling equilibria and profitable entry deterrence with positive probability, with only a dimension of private information by the incumbent but with two IS signals correlated with price, that provide additional (probabilistic) information about the incumbent type's. Surprisingly, the incumbent is better off with the use of a IS of intermediate precision

by the entrant because signals help the entrant smooth her best response. Thus, the incumbent can deter profitable entry in a pooling sequential equilibrium with probability α if he is of type L and with probability $(1-\alpha)$ if he is of type H .

5. Conclusions.

In this paper we analyzed industrial espionage when a potential entrant, E , does not observe the outcome of the R&D project carried out by an incumbent monopolist with the aim to reduce his cost of production and deter E from entering the market. E develops an Intelligence System (IS) of precision α that allows her to gather noisy information about the cost structure of M . Based on this information and the price that M charges for his product. E decides whether or not to enter the market. We assumed that α is exogenously given and commonly known by both firms.

We showed that the separating equilibria of our model are not affected by the spying activity of E . This is not very surprising since in a separating equilibrium E identifies the type of M with or without the use of the IS. The same result is obtained for pooling equilibria if the precision α of the IS is sufficiently low to affect the entrant's decision of staying out. If α is very accurate, then pooling equilibrium does not exist. For intermediate values of α we find that pooling equilibrium exists even under profitable entry and E enters the market if the IS tells her the cost of M is high. From this point of view, spying on incumbent firms increases market competition with high probability.

An interesting suggestion for further research might be to analyze the more realistic scenario where α is the private information of E .

6. References

Albaek, S. and Overgaard, P.B. (1992a) "Advertising and Pricing to Deter or Accommodate Entry When Demand Is Unknown: Comment", *International journal of Industrial Organization*, 12:1, 83-87.

Albaek, S. and Overgaard, P.B. (1992b) "Upstream Pricing and Advertising Signal Downstream Demand", *Journal of Economics and Management Strategy*, 1:4, 677-698.

- Bagwell, K. (1992) "A Model of Competitive Limit Pricing", *Journal of Economics and Management Strategy*, 1:4, 585-606.
- Bagwell, K. (2007) "Signalling and Entry Deterrence: A Multidimensional Analysis", *The RAND Journal of Economics*, 38:3, 670-697.
- Bagwell, K. and Ramey, G. (1988) "Advertising and Limit Pricing", *The RAND Journal of Economics*, 19:1, 59-71.
- Bagwell, K. and Ramey, G. (1990) "Advertising and Pricing to Deter or Accommodate Entry When Demand Is Unknown", *International Journal of Industrial Organization*, 8:1, 93-113.
- Bagwell, K. and Ramey, G. (1991) "Oligopoly Limit Pricing", *The RAND Journal of Economics*, 22:2, 155-172.
- Barrachina, A., Tauman, Y. and Urbano, A. (2014) "Entry and espionage with noisy signals", *Games and Economic Behavior*, 83, 127-146.
- Harrington, J.E. (1986) "Limit Pricing When the Potential Entrant Is Uncertain of Its Cost Function", *Econometrica*, 54:2, 429-437.
- Harrington, J.E. (1987) "Oligopolistic Entry Deterrence under Incomplete Information", *The RAND Journal of Economics*, 18:2, 211-231.
- Ho, S.J. (2007) "An Economic Analysis of Military Intelligence", *Defense and Peace Economics*, 18:6, 485-493.
- Ho, S.J. (2008) "Extracting the Information: Espionage with Double Crossing", *Journal of Economics*, 93:1, 31-58.
- Linnemer, L. (1998) "Entry Deterrence, Product Quality: Price and Advertising as Signals", *Journal of Economics and Management Strategy*, 7:4, 615-645.
- Matthews, S.A. and Mirman, L. (1983) "Equilibrium Limit Pricing: The effect of private information and stochastic demand", *Econometrica*, 51:4, 981-996.

Milgrom, P. (1981) "Good News and Bad News: Representation Theorems and Applications," *Bell Journal of Economics*, 12, 380-391.

Milgrom, P. and Roberts, J. (1982) "Limit Pricing and Entry under Incomplete Information: an Equilibrium Analysis," *Econometrica*, 50:2, 443-459.

Nasheri, H. (2005) *Economic Espionage and Industrial Spying*. Cambridge: Cambridge University Press.

Perea, A. and Swinkels, J. (1999) "Selling Information in Extensive Form Games", Working Paper. <http://e-archivo.uc3m.es/bitstream/10016/6151/1/we994418.PDF> (accessed June 2012).

Sakai, Y. (1985) "The Value of Information in a Simple Duopoly Model", *Journal of Economic Theory*, 36:1, 36-54.

Solan E. and Yariv, L. (2004) "Games with Espionage", *Games and Economic Behavior*, 47:1, 172-199.

7. Appendix

Proof of Lemma 1. (i) $\Pi_L(p) - \Pi_H(p) = C_H(Q(p)) - C_L(Q(p))$

$$\frac{\partial}{\partial p} [\Pi_L(p) - \Pi_H(p)] = Q'(p) [C'_H(Q(p)) - C'_L(Q(p))]$$

By Assumptions 4 and 5 the right hand side of the above expression is negative. (ii)

$$p_L^M q_L^M - C_L(q_L^M) \geq p_H^M q_H^M - C_L(q_H^M)$$

$$p_H^M q_H^M - C_H(q_H^M) \geq p_L^M q_L^M - C_H(q_L^M)$$

Adding the two inequalities we have

$$C_H(q_L^M) - C_L(q_L^M) \geq C_H(q_H^M) - C_L(q_H^M)$$

By Assumption 4 we have that $q_L^M \geq q_H^M$ and hence $p_L^M \leq p_H^M$.

Let us show that $p_L^M < p_H^M$. If not, then $p_L^M = p_H^M$. Since the First Order Condition (FOC) for M of type t is

$$\frac{\partial \Pi_t}{\partial p}(Q(p)) = 0 \leftrightarrow C'_t(Q(p)) = p + \frac{Q(p)}{Q'(p)}$$

the solution does not depend on t , namely $C'_L(Q(p_L^M)) = C'_H(Q(p_L^M))$. But this contradicts Assumption 4. (iii) By Assumption 3,

$$\Pi_L(p_L^M) - D_L > \Pi_H(p_H^M) - D_H$$

Note that $D_L = \Pi_L(p_0)$ and $D_H = \Pi_H(\hat{p})$. Hence this inequality can be written as

$$\Pi_L(p_L^M) - \Pi_L(p_0) > \Pi_H(p_H^M) - \Pi_H(\hat{p})$$

Thus,

$$\Pi_L(p_L^M) - \Pi_H(p_L^M) + \underbrace{\Pi_H(p_L^M) - \Pi_H(p_H^M)}_{<0} > \Pi_L(p_0) - \Pi_H(\hat{p})$$

Hence,

$$\Pi_L(p_L^M) - \Pi_H(p_L^M) > \Pi_L(p_0) - \Pi_H(\hat{p}) \quad (\text{A1})$$

Since $p_0 \leq p_L^M$, we have by section (i) of Lemma 1

$$\Pi_L(p_0) - \Pi_H(p_0) > \Pi_L(p_L^M) - \Pi_H(p_L^M)$$

This together with (A1) imply that

$$\Pi_H(\hat{p}) > \Pi_H(p_0)$$

But $p_0 < p_H^M$ and $\hat{p} < p_H^M$ and by Assumption 2 $p_0 < \hat{p}$. ■

Proof of Proposition 1. It remains to be shown that if $p_L \notin [p_0, \min(p_L^M, \hat{p})]$, then

$p_L \notin SSE_L$. Let $Q = [p_0, \min(p_L^M, \hat{p})]$.

Case A: $p_L^M \leq \hat{p}$.

Subcase A.1. Suppose that $p_L^M \leq \hat{p} < p_h$. There is no separating equilibrium in this case since by (14), in the main text, $p_h \leq \hat{p}$, a contradiction.

Subcase A.2. Suppose that $p_L^M \leq p_h \leq \hat{p}$. Then by Assumption 2 L is better off choosing $p_L = p_L^M$ and $p_L^M \in Q$.

Subcase A.3. Suppose that $p_h < p_L^M \leq p_l$. Since $p_L \leq p_h < p_L^M$, by Assumption 2 L is better off choosing $p_L = p_h < p_L^M$. From the incentive compatibility constraint of L we have

$$\Pi_L(p_h) + \Pi_L(p_L^M) \geq \Pi_L(p_L^M) + \alpha \Pi_L(p_L^M) + (1 - \alpha) D_L \quad (\text{A2})$$

or

$$\Pi_L(p_h) \geq \alpha \Pi_L(p_L^M) + (1-\alpha) D_L = \Pi_L(\tilde{p}_L(\alpha))$$

Thus $p_h \geq \tilde{p}_L(\alpha)$. Consequently,

$$p_0 < \tilde{p}_L(\alpha) \leq p_L = p_h < p_L^M$$

and hence $p_L \in Q$.

Subcase A.4. Suppose that $p_l < p_L^M$. Similarly to the previous case, L is better off choosing $p_L = p_h < p_L^M$. In order for L not to deviate, (19), in the main text, must hold.

Equivalently $p_h \geq p_0$. Hence

$$p_0 \leq p_L = p_h < p_L^M$$

and $p_L \in Q$.

Case B: $\hat{p} < p_L^M$.

Subcase B.1. Suppose that $p_L^M \leq p_h$. There is no separating equilibrium in this case since by (14) in the main text $p_h \leq \hat{p}$, a contradiction.

Subcase B.2. Suppose that $p_h < p_L^M \leq p_l$. Then, L is better off choosing $p_L = p_h$.

By (14), in order for H not to deviate, $p_h \leq \hat{p}$ must hold. By (A2), L has no incentive to deviate if $p_h \geq \tilde{p}_L(\alpha)$. Consequently,

$$p_0 < \tilde{p}_L(\alpha) \leq p_L = p_h \leq \hat{p}$$

and $p_L \in Q$.

Subcase B.3. Suppose that $p_l < p_L^M$. Again, L is better off choosing $p_L = p_h$ and $p_h \leq \hat{p}$ must hold. To guarantee that L has no incentive to deviate, $p_h \geq p_0$ must hold (see (19) in the main text). Hence

$$p_0 \leq p_L = p_h \leq \hat{p}$$

and $p_L \in Q$. ■

Proof of Proposition 2. Since in a separating equilibrium E can identify the type of M, in any separating equilibrium \bar{p} should satisfy $p_L \leq \bar{p} < p_H$. The H-type monopoly, knowing that entry will occur, is better off choosing the price p_H^M . That is, $p_H = p_H^M$. Hence, in a limit price separating equilibrium $p_L \neq p_L^M$.

Lemma A1. Suppose that

(1) $p_L \neq p_L^M$. Then, $\bar{p} < p_L^M$ and $p_L = \bar{p}$ (separating limit price equilibrium)

(2) $p_L = p_L^M \leq \bar{p}$, then $p_L = p_L^M \leq \bar{p}$.

Proof: (1) Suppose to the contrary that $p_L^M \leq \bar{p}$. Then, by the definition of \bar{p} , E stays out when observing the price p_L^M . But then L is better off deviating from p_L to p_L^M , a contradiction. Let us next show that $p_L = \bar{p}$. Clearly $p_L \leq \bar{p}$ (since when E observes p_L , she stays out). Suppose to the contrary that $p_L < \bar{p}$. Since $\bar{p} < p_L^M$, by Assumption 2, L is better off increasing his price from p_L to \bar{p} , a contradiction. Thus, $p_L = \bar{p}$.

(2) As in Proposition 1, we have to check that neither H nor L have incentives to deviate. In order to H not to deviate to p_L , the next inequality must hold

$$\Pi_H(p_H^M) + D_H \geq \Pi_H(p_L) + \Pi_H(p_H^M)$$

Equivalently,

$$\Pi_H(p_L) \leq D_H$$

or $p_L \leq \hat{p}$. Let us next check the incentive compatible constraint for L. In order to L not to deviate to p_L^M , the next inequality must hold

$$\Pi_L(p_L) + \Pi_L(p_L^M) \geq \Pi_L(p_L^M) + D_L$$

Equivalently,

$$\Pi_L(p_L) \geq D_L$$

or $p_L \geq p_0$. ■

Proof of Proposition 3. Let $A_l = \{\alpha \mid \Pi_E(l \mid \alpha) \leq 0\}$ and $A_h = \{\alpha \mid \Pi_E(h \mid \alpha) \leq 0\}$.

(1) **Suppose that** $\mu \Delta_E(H) + (1 - \mu) \Delta_E(L) < 0$. In this case $\bar{\alpha}_l < \frac{1}{2} < \bar{\alpha}_h < 1$. Hence, $\alpha > \bar{\alpha}_l$ and by Corollary 1, $\alpha \in A_l \forall \alpha, \frac{1}{2} < \alpha < 1$. Namely, if the IS sends the signal l , E does not enter the market when observing the price p^* irrespective the precision α of the IS. The proof of Proposition 3 is split in two main cases a) $\frac{1}{2} < \alpha \leq \bar{\alpha}_h$ and b) $\bar{\alpha}_h < \alpha < 1$.

(1.a) We start by considering that $\frac{1}{2} < \alpha \leq \bar{\alpha}_h$. In this case $\alpha \in A_l \cap A_h$. Namely, E does not enter the market when observing the price p^* irrespective of the signal sent by the IS. Let us characterize the pooling equilibria in this case. We start with a lemma.

Lemma A2. Suppose that $\mu\Delta_E(H) + (1-\mu)\Delta_E(L) < 0$ and $\frac{1}{2} < \alpha \leq \bar{\alpha}_h$. Then in every pooling equilibrium

- (i) $p_H^M > p_h$
- (ii) $p^* \leq p_l$

Proof: (i) Suppose to the contrary that $p_H^M \leq p_h$. Then also $p_L^M < p_h$ and E stays out whether she observes p_L^M or p_H^M . Hence, both types of M will set their (different) monopoly prices, a contradiction.

(ii) Suppose to the contrary that $p^* > p_l$. Then at least one type of M has an incentive to deviate. Indeed if $p_L^M > p_l$ and $p^* = p_L^M$, the H-type monopoly is better off deviating to p_H^M or p_l depending on α and the parameters of the model. Similarly, if $p_L^M > p_l$ and $p^* = p_H^M$ the L-type monopoly is better off deviating to p_L^M or p_l depending on α and the parameters of the model. Finally, if $p_L^M \leq p_l$, at least one type of M has an incentive to deviate to his monopoly price, a contradiction. ■

Consider that $p_L^M < \hat{p}$. We prove next, (1.(i)), when $\frac{1}{2} < \alpha \leq \bar{\alpha}_h$:

Lemma A3. Suppose that $\mu\Delta_E(H) + (1-\mu)\Delta_E(L) < 0$, $\frac{1}{2} < \alpha \leq \bar{\alpha}_h$ and $p_L^M < \hat{p}$. Then $SPE = \emptyset$.

Proof: Suppose to the contrary that $p^* \in SPEP$ and suppose that p_h and p_l are the equilibrium thresholds of E. We consider six cases.

Case 1. Suppose that $p_L^M \leq p_h$. First note that in this case $p^* = p_L^M$. Next observe that the incentive compatibility constraint of H

$$\Pi_H(p_L^M) + \Pi_H(p_H^M) \geq \Pi_H(p_H^M) + D_H \quad (\text{A3})$$

requiring that H has no incentive to deviate to p_H^M , is equivalent to

$$\Pi_H(p_L^M) \geq D_H = \Pi_H(\hat{p})$$

and hence $p_L^M \geq \hat{p}$, a contradiction.

Case 2. Suppose that $p_h < p_L^M < p_H^M \leq p_l$. In this case $p^* \leq p_h$ (otherwise, by Lemma A2, $p^* \in (p_h, p_l]$ and at least one type of M has an incentive to deviate to his monopoly price). Therefore $p^* = p_h$ must hold. The incentive compatibility constraint of H is,

$$\Pi_H(p_h) + \Pi_H(p_H^M) \geq \Pi_H(p_H^M) + \alpha D_H + (1-\alpha)\Pi_H(p_H^M) \quad (\text{A4})$$

Equivalently,

$$\Pi_H(p_h) \geq \alpha D_H + (1-\alpha)\Pi_H(p_H^M) = \Pi_H(\hat{p}_H(\alpha))$$

or $\hat{p}_H(\alpha) \leq p_h < p_H^M$. But $p_L^M < \hat{p} < \hat{p}_H(\alpha)$ and in particular $p_L^M < p_h$, a contradiction.

Case 3. Suppose that $p^* \leq p_h < p_L^M \leq p_l < p_H^M$. In this case again $p^* = p_h$. The incentive compatibility constraint of H is

$$\Pi_H(p_h) + \Pi_H(p_H^M) \geq \Pi_H(p_H^M) + D_H \quad (\text{A5})$$

Equivalently,

$$\Pi_H(p_h) \geq D_H = \Pi_H(\hat{p})$$

or $\hat{p} \leq p_h$. But we deal with the case where $p_h < p_L^M < \hat{p}$, a contradiction.

Case 4. Suppose that $p_h < p_L^M \leq p_l < p_H^M$ and $p^* \in (p_h, p_l]$. In this case $p^* = p_L^M$. In order for H not to deviate from p_L^M to p_H^M , the inequality

$$\Pi_H(p_L^M) + \alpha D_H + (1-\alpha)\Pi_H(p_H^M) \geq \Pi_H(p_H^M) + D_H \quad (\text{A6})$$

should hold. Equivalently,

$$\alpha \leq \frac{\Pi_H(p_L^M) - D_H}{\Pi_H(p_H^M) - D_H} = \frac{\Pi_H(p_L^M) - \Pi_H(\hat{p})}{\Pi_H(p_H^M) - \Pi_H(\hat{p})} \equiv \delta < 0$$

a contradiction.

Case 5. Suppose that $p^* \leq p_h < p_l < p_L^M < p_H^M$. In this case $p^* = p_h$ and (A5) guarantees that H has no incentive to deviate from p^* to p_H^M . By (A5), $\hat{p} \leq p_h < p_L^M$, a contradiction.

Case 6. Suppose that $p_h < p_l < p_L^M < p_H^M$ and $p^* \in (p_h, p_l]$. Clearly in this case $p^* = p_l$ and E follows the signal sent by the IS. In order for H not to deviate from p_l to p_H^M , the inequality

$$\Pi_H(p_l) + \alpha D_H + (1-\alpha)\Pi_H(p_H^M) \geq \Pi_H(p_H^M) + D_H \quad (\text{A7})$$

must hold. Equivalently,

$$\Pi_H(p_l) \geq \alpha\Pi_H(p_H^M) + (1-\alpha)D_H = \Pi_H(\tilde{p}_H(\alpha))$$

and $\tilde{p}_H(\alpha) \leq p_l < p_H^M$. But $p_L^M < \hat{p} < \tilde{p}_H(\alpha)$, a contradiction. We conclude that if $p_L^M < \hat{p}$, then $SPE = \emptyset$. ■

We next deal with the case (1 (iii.1)), where $\hat{p} < p_L^M$. Lemmas A4 and A5 below prove this result. We need to show that $SPEP = \{p^* \mid \hat{p} \leq p^* \leq p_L^M\}$ for all α , $\frac{1}{2} < \alpha \leq \bar{\alpha}_h$.

Lemma A4. Suppose that $\mu\Delta_E(H) + (1-\mu)\Delta_E(L) < 0$, $\frac{1}{2} < \alpha \leq \bar{\alpha}_h$ and $\hat{p} < p_L^M$. Then

$$\{p^* \mid \hat{p} \leq p^* \leq p_L^M\} \subseteq SPEP$$

Proof: We start by showing that $p_L^M \in SPEP$. Let p_h and p_l be such that

$$p^* = p_h = p_L^M < p_l < p_H^M \quad (\text{A8})$$

Clearly L is better off with p_L^M and has no incentive to deviate. The two incentive compatibility constraints of H in this case are:

(i) H has no incentive to deviate from p_L^M to p_l . Namely,

$$\Pi_H(p_L^M) + \Pi_H(p_H^M) \geq \Pi_H(p_l) + \alpha D_H + (1-\alpha)\Pi_H(p_H^M) \quad (\text{A9})$$

Equivalently,

$$\Pi_H(p_l) - \Pi_H(p_L^M) \leq \alpha [\Pi_H(p_H^M) - D_H]$$

(ii) H has no incentive to deviate from p_L^M to p_H^M if (A3) holds.

Since $\hat{p} < p_L^M$, (A3) holds. As for (A9), it holds for every $\frac{1}{2} < \alpha < 1$, provided that p_l is sufficiently close to p_L^M . Hence (A8) for p_l sufficiently close to p_L^M , guarantees that, for all $\frac{1}{2} < \alpha < 1$, $p_L^M \in SPEP$.

Next let p_h and p_l be such that

$$\hat{p} \leq p^* = p_h < p_l < p_L^M < p_H^M \quad (\text{A10})$$

The incentive compatibility constraints of L in this case are two:

- (i) L has no incentive to deviate from p_h to p_l .

$$\Pi_L(p_h) + \Pi_L(p_L^M) \geq \Pi_L(p_l) + \alpha \Pi_L(p_L^M) + (1-\alpha)D_L \quad (\text{A11})$$

Equivalently,

$$\Pi_L(p_l) - \Pi_L(p_h) \leq (1-\alpha) [\Pi_L(p_L^M) - D_L]$$

- (ii) L has no incentive to deviate from p_h to p_L^M .

$$\Pi_L(p_h) + \Pi_L(p_L^M) \geq \Pi_L(p_L^M) + D_L \quad (\text{A12})$$

Equivalently,

$$\Pi_L(p_h) \geq D_L = \Pi_L(p_0)$$

or $p_0 \leq p_h$.

The two incentive compatibility constraints of H are the one given by (A5) and

$$\Pi_H(p_h) + \Pi_H(p_H^M) \geq \Pi_H(p_l) + \alpha D_H + (1-\alpha) \Pi_H(p_H^M) \quad (\text{A13})$$

Equivalently,

$$\Pi_H(p_l) - \Pi_H(p_h) \leq \alpha [\Pi_H(p_H^M) - D_H]$$

Inequalities (A5) and (A12) imply that $\hat{p} \leq p_h$ and $p_0 \leq p_h$ respectively. By Lemma 1, $\hat{p} > p_0$. Hence, $\hat{p} \leq p_h$, which is consistent with (A10). On the other hand, (A11) and

(A13) hold for every $\frac{1}{2} < \alpha < 1$ provided that p_h is sufficiently close to p_l . Hence

(A10) for $0 < p_l - p_h$ sufficiently small, guarantees that, for all $\frac{1}{2} < \alpha < 1$, $p^* \in \text{SPEP}$,

and the proof of Lemma A4 is completed. ■

Lemma A5. Suppose that $\mu \Delta_E(H) + (1-\mu) \Delta_E(L) < 0$ and $\frac{1}{2} < \alpha \leq \bar{\alpha}_h$. Then, $\text{SPEP} \subseteq [\hat{p}, p_L^M]$.

Proof: Let $p^* \in \text{SPEP}$. By Lemma A2, $p^* \leq p_l$ and $p_H^M > p_h$. Let $R = [\hat{p}, p_L^M]$.

The relevant cases are

Case 1. Suppose that $p_L^M \leq p_h$. Then $p^* = p_L^M \in R$.

Case 2. Suppose that $\hat{p} < p_h < p_L^M < p_H^M \leq p_l$. Similarly to case 2 of Lemma A3, $p^* = p_h$. By the incentive compatibility constraint of H given by (A4), $\hat{p} < \hat{p}_H(\alpha) \leq p^* = p_h < p_L^M$. Hence $p^* \in R$.

Case 3. Suppose that $p_h \leq \hat{p} < p_L^M < p_H^M \leq p_l$. Similarly to the previous case $\hat{p}_H(\alpha) \leq p^* = p_h$. Hence, no pooling equilibrium exists in this case since $p_h \leq \hat{p} < \hat{p}_H(\alpha)$.

Case 4. Suppose that $\hat{p} \leq p^* \leq p_h < p_L^M \leq p_l < p_H^M$. Then clearly $p^* \in R$.

Case 5. Suppose that $p^* \leq p_h < \hat{p} < p_L^M \leq p_l < p_H^M$. Similarly to the previous case $p^* = p_h$, and by (A5) $\hat{p} \leq p_h$, a contradiction. Hence, there exists no pooling equilibrium in this case.

Case 6. Suppose that $p_h < p_L^M \leq p_l < p_H^M$ and $p^* \in (p_h, p_l]$. Clearly in this case $p^* = p_L^M$ and $p_L^M \in SPEP$.

Case 7. Suppose that $\hat{p} \leq p^* \leq p_h < p_l < p_L^M < p_H^M$. Then $p^* \in R$.

Case 8. Suppose that $p^* \leq p_h < \hat{p} < p_l < p_L^M < p_H^M$. Similarly to case 5 above, there is no pooling equilibrium in this case.

Case 9. Suppose that $p_h < \hat{p} < p_l < p_L^M < p_H^M$ and $p^* \in (p_h, p_l]$. Clearly in this case $p^* = p_l$. From the incentive compatibility constraint of H given by (A7), $\hat{p} < \tilde{p}_H(\alpha) \leq p^* = p_l < p_L^M$. Hence $p^* \in R$.

Case 10. Suppose that $p_h < p_l \leq \hat{p} < p_L^M < p_H^M$ and $p^* \in (p_h, p_l]$. Similarly to the previous case $\tilde{p}_H(\alpha) \leq p^* = p_l$. Hence, no pooling equilibrium exists in this case since $p_l \leq \hat{p} < \tilde{p}_H(\alpha)$.

The above 10 cases prove that if $p^* \in SPEP$, then $p^* \in R$, as claimed. ■

Now, let us show by the following lemmas A6 and A7 the first part of point (1. (ii)) of Proposition 3, i.e. that if $p_L^M = \hat{p}$, then $SPEP = \{p_L^M\}$ for all $\frac{1}{2} < \alpha \leq \bar{\alpha}_h$.

Lemma A6. Suppose that $\mu\Delta_E(H) + (1-\mu)\Delta_E(L) < 0$, $\frac{1}{2} < \alpha \leq \bar{\alpha}_h$ and $p_L^M = \hat{p}$.

Then $p_L^M \in SPEP$.

Proof: Let p_h and p_l be such that (A8) holds. The incentive compatibility constraints of H are given by (A3) and (A9). Clearly (A3) holds since $p_L^M = \hat{p}$. But also (A9) holds for every $\frac{1}{2} < \alpha < 1$, provided that p_l is sufficiently close to p_L^M . Hence, $p_L^M \in SPEP$ for all $\frac{1}{2} < \alpha < 1$ is guaranteed by (A8) with p_l sufficiently close to p_L^M . ■

Lemma A7. Suppose that $\mu\Delta_E(H) + (1-\mu)\Delta_E(L) < 0$, $\frac{1}{2} < \alpha \leq \bar{\alpha}_h$, $p_L^M = \hat{p}$ and $p^* \in SPEP$. Then $p^* = p_L^M$.

Proof: We consider the same six cases as in the proof of Lemma A3.

Case 1. Suppose that $p_L^M \leq p_h$. Then $p^* = p_L^M$, as claimed.

Case 2. Suppose that $p_h < p_L^M < p_H^M \leq p_l$. Similarly to case 2 of Lemma A3, $p^* = p_h$. By (A4), $\hat{p} < \hat{p}_H(\alpha) \leq p_h < p_H^M$ must hold. But $p_h < p_L^M = \hat{p}$, a contradiction. Hence, there is no pooling equilibrium in this case.

Case 3. Suppose that $p^* \leq p_h < p_L^M \leq p_l < p_H^M$. In this case again $p^* = p_h$. The incentive compatibility constraint of H is given by (A5) and implies $\hat{p} \leq p_h$. But in this case $p_h < p_L^M = \hat{p}$. Consequently, no pooling equilibrium exists in this case.

Case 4. Suppose that $p_h < p_L^M \leq p_l < p_H^M$ and $p^* \in (p_h, p_l]$. In this case $p^* = p_L^M \in SPEP$, as claimed.

Case 5. Suppose that $p^* \leq p_h < p_l < p_L^M < p_H^M$. In this case $p^* = p_h$ and H has no incentive to deviate from p^* to p_H^M if (A5) holds, or equivalently, $\hat{p} \leq p_h < p_L^M$, a contradiction. Hence, there is no pooling equilibrium in this case either.

Case 6. Suppose that $p_h < p_l < p_L^M < p_H^M$ and $p^* \in (p_h, p_l]$. Clearly in this case $p^* = p_l$. From the incentive compatibility constraint of H given by (A7), $\tilde{p}_H(\alpha) \leq p_l$ must hold. But $p_L^M = \hat{p} < \tilde{p}_H(\alpha)$. Consequently, no pooling equilibrium exists in this case. ■

Lemmas A6 and A7 establish the first part of point (1, (ii)) of the proposition.

(1.b) Consider now that $\bar{\alpha}_h < \alpha < 1$. In this case $\alpha \in A_l \setminus \bar{A}_h$. Namely, E enters the market when observing the price p^* if the IS sends the signal h and does not enter if

the IS sends the signal l . Hence, accordingly to the strategy of E defined in Lemma 4,

$$p_h < p^* \leq p_l.$$

Let us find pooling equilibria in this case. First,

Lemma A8. Suppose that $\mu\Delta_E(H) + (1-\mu)\Delta_E(L) < 0$ and $\bar{\alpha}_h < \alpha < 1$. Then, in every pooling equilibrium $p_L^M > p_h$ and $p_H^M > p_l$.

Proof: Suppose to the contrary that $p_L^M \leq p_h$ or $p_H^M \leq p_l$. Then, at least one type of M has an incentive to deviate to his monopoly price. ■

The next lemma completes the proof of points (1 (i)) and (1 (ii)) of the Proposition.

Lemma A9. Suppose that $\mu\Delta_E(H) + (1-\mu)\Delta_E(L) < 0$, $\bar{\alpha}_h < \alpha < 1$ and $p_L^M \leq \hat{p}$. Then $SPE = \emptyset$.

Proof: Suppose to the contrary that $p^* \in SPEP$. We consider two cases.

Case 1. Suppose that $p_h < p_L^M \leq p_l < p_H^M$. Note that in this case $p^* = p_l = p_L^M$. In order for H not to deviate from p_L^M to p_H^M , (A6) should hold. Equivalently $\alpha \leq \delta \leq 0$, a contradiction.

Case 2. Suppose that $p_h < p_l < p_L^M < p_H^M$. Note that in this case $p^* = p_l$. H has no incentive to deviate from p_l to p_H^M if (A7) holds. Equivalently, $p_l \geq \tilde{p}_H(\alpha)$. Since $p_l < p_L^M$, $p_L^M > \tilde{p}_H(\alpha)$ must hold. Equivalently, $\alpha < \delta \leq 0$, a contradiction. ■

Let us prove next that if $\bar{\alpha}_h < \alpha \leq \delta$, then $SPEP = [\max(\tilde{p}_H(\alpha), \hat{p}_L(\alpha)), p_L^M]$. (point (1 (iii.2)) of the Proposition).

Lemma A10. Suppose that $\mu\Delta_E(H) + (1-\mu)\Delta_E(L) < 0$, $\bar{\alpha}_h < \alpha < 1$ and $\hat{p} < p_L^M$. Then

$$SPEP = \left\{ p^* \mid \max(\tilde{p}_H(\alpha), \hat{p}_L(\alpha)) \leq p^* \leq p_L^M \right\}$$

and this set is non-empty if $\delta > \bar{\alpha}_h$ and for all α , $\bar{\alpha}_h < \alpha \leq \delta$.

Proof: We start by showing that $p_L^M \in SPEP$. Let p_h and p_l be s.t.

$$p_h < p^* = p_L^M = p_l < p_H^M \quad (A14)$$

In order for H not to deviate from p_L^M to p_H^M , (A6) should hold. Equivalently $\alpha \leq \delta$,

where $0 < \delta < 1$ since $p_L^M > \hat{p}$. H has no incentive to deviate from p_L^M to p_h , if

$$\Pi_H(p_L^M) + \alpha D_H + (1-\alpha)\Pi_H(p_H^M) \geq \Pi_H(p_h) + \Pi_H(p_H^M) \quad (A15)$$

holds. Equivalently,

$$\Pi_H(p_L^M) - \Pi_H(p_h) \geq \alpha(\Pi_H(p_H^M) - D_H)$$

The incentive compatibility constraint of L is given by

$$\Pi_L(p_L^M) + \alpha\Pi_L(p_L^M) + (1-\alpha)D_L \geq \Pi_L(p_h) + \Pi_L(p_L^M) \quad (\text{A16})$$

Equivalently,

$$\Pi_L(p_L^M) - \Pi_L(p_h) \geq (1-\alpha)(\Pi_L(p_L^M) - D_L)$$

Note that (A15) and (A16) hold for p_h sufficiently small. Hence (A14), p_h sufficiently small and for $\delta > \bar{\alpha}_h$, guarantees that, for all $\bar{\alpha}_h < \alpha \leq \delta$, $p_L^M \in \text{SPEP}$.

Next let p_h and p_l be such that

$$p_h < \max(\tilde{p}_H(\alpha), \hat{p}_L(\alpha)) \leq p^* = p_l < p_L^M < p_H^M \quad (\text{A17})$$

H has no incentive to deviate from p_l to p_H^M if (A7) holds. Equivalently, $p_l \geq \tilde{p}_H(\alpha)$.

Since $p_l < p_L^M$, $p_L^M > \tilde{p}_H(\alpha)$ must hold. Equivalently, $\alpha < \delta$. Note that $0 < \delta < 1$ since $p_L^M > \hat{p}$. In order for H not to deviate from p_l to p_h ,

$$\Pi_H(p_l) + \alpha D_H + (1-\alpha)\Pi_H(p_H^M) \geq \Pi_H(p_h) + \Pi_H(p_H^M) \quad (\text{A18})$$

should hold. Equivalently,

$$\Pi_H(p_l) - \Pi_H(p_h) \geq \alpha(\Pi_H(p_H^M) - D_H)$$

Next, let us consider the two incentive compatibility constraints of L.

(i) In order for L not to deviate from p_l to p_L^M ,

$$\Pi_L(p_l) + \alpha\Pi_L(p_L^M) + (1-\alpha)D_L \geq \Pi_L(p_L^M) + D_L \quad (\text{A19})$$

should hold. Equivalently, $p_l \geq \hat{p}_L(\alpha)$.

(ii) In order for L not to deviate from p_l to p_h ,

$$\Pi_L(p_l) + \alpha\Pi_L(p_L^M) + (1-\alpha)D_L \geq \Pi_L(p_h) + \Pi_L(p_L^M) \quad (\text{A20})$$

should hold. Equivalently,

$$\Pi_L(p_l) - \Pi_L(p_h) \geq (1-\alpha)(\Pi_L(p_L^M) - D_L)$$

(A7) and (A19) imply that $\max(\tilde{p}_H(\alpha), \hat{p}_L(\alpha)) \leq p^* = p_l < p_L^M$, which is consistent with (A17), but it needs $\delta > \bar{\alpha}_h$ and $\bar{\alpha}_h < \alpha < \delta$. Note that (A18) and (A20) hold for p_h

sufficiently small. Hence, (A17) for p_h sufficiently small and $\delta > \bar{\alpha}_h$, guarantees that, for all $\bar{\alpha}_h < \alpha < \delta$, $p^* \in SPEP$. ■

Notice that the above implies (1(iii.3)), i.e., for $\alpha > \delta$, $SPE = \emptyset$.

(2) Suppose now that $\mu\Delta_E(H) + (1-\mu)\Delta_E(L) > 0$. Note that in this case $\bar{\alpha}_h < 1/2 < \bar{\alpha}_l < 1$. Hence $\alpha > \bar{\alpha}_h$ and $\alpha \notin A_h$, $\forall \alpha$, $1/2 < \alpha < 1$. Namely, if the IS sends the signal h , E enters the market when observing the price p^* irrespective the precision α of the IS.

(2.a) Consider that $1/2 < \alpha < \bar{\alpha}_l$. In this case $\alpha \notin A_l \cup A_h$. Namely, E enters the market when observing the price p^* irrespective of the signal sent by the IS and, therefore, both H and L should select the prices p_H^M and p_L^M , respectively. Since $p_L^M < p_H^M$, no pooling equilibrium exists in this case, i.e., for $\alpha < \bar{\alpha}_l$, $SPE = \emptyset$ (point (2.(ii.1))).

(2.b) Suppose now that $\bar{\alpha}_l \leq \alpha < 1$. In this case $\alpha \in A_l \setminus \bar{A}_h$. Namely, E enters the market when observing the price p^* if the IS sends the signal h and does not enter if the IS sends the signal l . In particular, if $\hat{p} < p_L^M$, then

$$SPEP = \left\{ p^* \mid \max(\tilde{p}_H(\alpha), \hat{p}_L(\alpha)) \leq p^* \leq p_L^M \right\}$$

and this set is non-empty if $\delta \geq \bar{\alpha}_l$ and for all α , $\bar{\alpha}_l \leq \alpha \leq \delta$. This proves case (2 (ii.2)).

As above, notice that (2(ii.3)), i.e., for $\alpha > \delta$, $SPE = \emptyset$ is satisfied.

As in the proof of (1 (i)) and (1 (ii)) (see lemma A3 and lemmas 6-9, above), if $p_L^M \leq \hat{p}$, then $SPE = \emptyset$, which proves case (2 (i))

Finally, notice that (2(ii.1)) and (2(ii.3)) prove the first part of (3), while (1(iii.3)) and (1(iii.1)) prove the second part. ■

Proof of Proposition 4. In a pooling equilibrium $p_H = p_L = p^*$. Suppose first that $\mu\Delta_E(H) + (1-\mu)\Delta_E(L) > 0$. Then E will enter when observing the price p^* . Hence, both H and L should select the prices p_H^M and p_L^M respectively. Since $p_L^M < p_H^M$, at least one of them deviates from p^* and pooling equilibrium does not exist.

Suppose next that $\mu\Delta_E(H) + (1-\mu)\Delta_E(L) < 0$. This means that E stays out when observing p^* . Hence, according to the entrant's decision rule $p^* \leq \bar{p}$.

Lemma A11. $p^* = \bar{p} \leq p_L^M$.

Proof: Let us first show that $\bar{p} < p_H^M$. Note that if $\bar{p} \geq p_H^M$, then also $p_L^M \leq \bar{p}$ (since $p_L^M < p_H^M$) and hence E stays out if she observes either p_L^M or p_H^M . Consequently, both L and H are best off choosing their monopoly price and at least one of them benefits from his deviation to his monopoly price, a contradiction. Let us next show that $p^* = \bar{p}$. If not, then $p^* < \bar{p} < p_H^M$ and by Assumption 2 H is better off deviating from p^* to \bar{p} .

Finally, we claim that $\bar{p} \leq p_L^M$. If not, then $p_L^M < p^* = \bar{p}$ and L is better off deviating from p^* to p_L^M . For p^* to be the pooling equilibrium price, neither L nor H must have incentive to deviate. In order to H not to deviate to his monopoly price, the next inequality must hold,

$$\Pi_H(p^*) + \Pi_H(p_H^M) \geq \Pi_H(p_H^M) + D_H$$

Equivalently, $\Pi_H(p^*) \geq D_H$, or $p^* \geq \hat{p}$.

In order to L not to deviate, the next inequality must hold,

$$\Pi_L(p^*) + \Pi_L(p_L^M) \geq \Pi_L(p_L^M) + D_L$$

Equivalently, $\Pi_L(p^*) \geq D_L$, or $p^* \geq p_0$. ■